Honors Physics
Unit 3: Application of Forces

Slides

Unit 3: Application of Forces

- Universal gravitation
- Weight/gravitational fields
- Free fall
- Air resistance
- Projectile motion
- Uniform circular motion
- Friction

Universal Gravitation

Gravity

Gravity is an attractive force (pulling force) between any two objects that have mass.

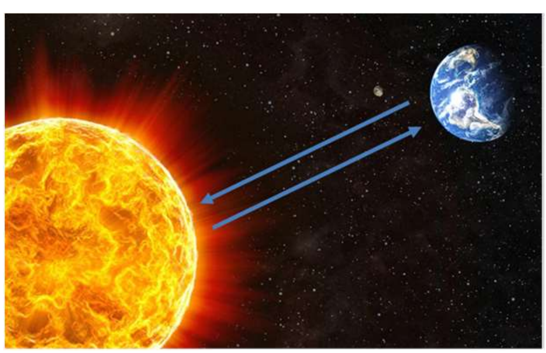
Near Earth's surface:

- Earth's gravity pulls all things toward the center of the Earth
- The more massive the object, the greater the strength of gravity exerted by the Earth

GRAVITY IS UNIVERSAL

- All matter has gravity (because it has mass).
- All matter in the universe attracts all other matter in the universe regardless of mass and distance apart.
- Gravity is everywhere and is always acting upon all objects, despite in most cases being very weak.

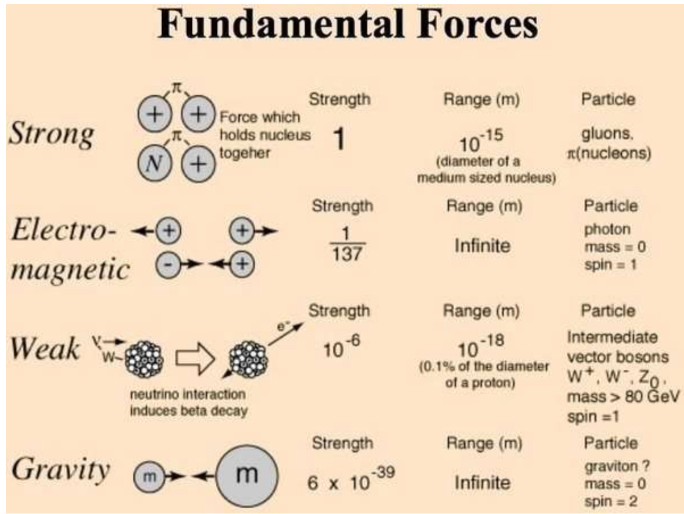
Gravity is an **action-at-a-distance force**. Earth's gravity (for example) has influence through space. Earth's gravity can weakly affect objects millions of km away from Earth.



The sun and the Earth are pulling on each other with gravity force, but not touching.

Gravity is a Fundamental Force

- Fundamental forces: gravity, electromagnetic force, strong force, weak force
- Gravity is by far the weakest of the four



Newton's Law of Universal Gravitation (In Words)

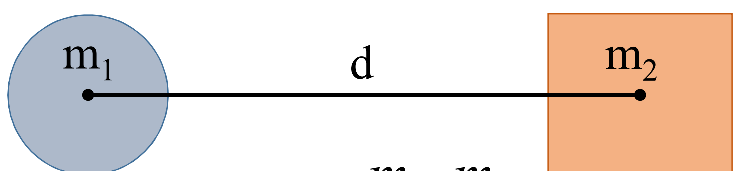
The attractive force due to gravity is proportional to the **product of the masses** of the attracting objects and inversely proportional to the **distance squared** between the objects.

Newton's Law of Universal Gravitation (Equation Form)

$$F_g = G \cdot \frac{m_1 \cdot m_2}{d^2}$$

m_1 = mass of object 1 (kg)
m_2 = mass of object 2 (kg)
d = distance between objects (m)
G = Universal gravity constant

$$G = 6.67 \times 10^{-11} N \frac{m^2}{kg^2}$$



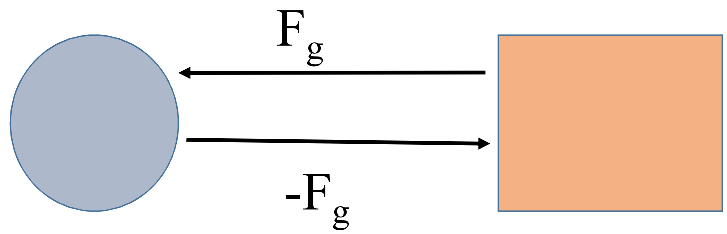
$$F_g = G \cdot \frac{m_1 \cdot m_2}{d^2}$$

Masses of the objects are multiplied in the numerator
- Greater the masses, greater the force
- Lesser the masses, lesser the force

Distance between the objects is in the denominator (squared)
- Greater the distance, lesser the force
- Lesser the distance, greater the force

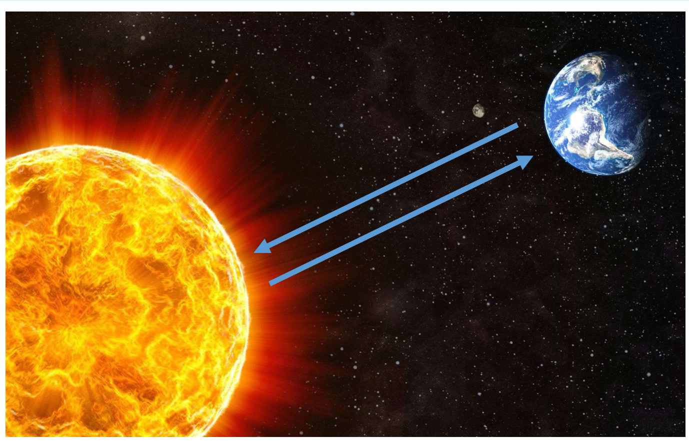
Gravitational attraction is mutual

- Obeys Newton's 3rd Law (equal and opposite)
- The force of one object attracting is the other is equal in magnitude and opposite in direction to the force of the other.



Force of blue circle pulling on red square is equal in magnitude and opposite in direction to the red square pulling on blue circle.

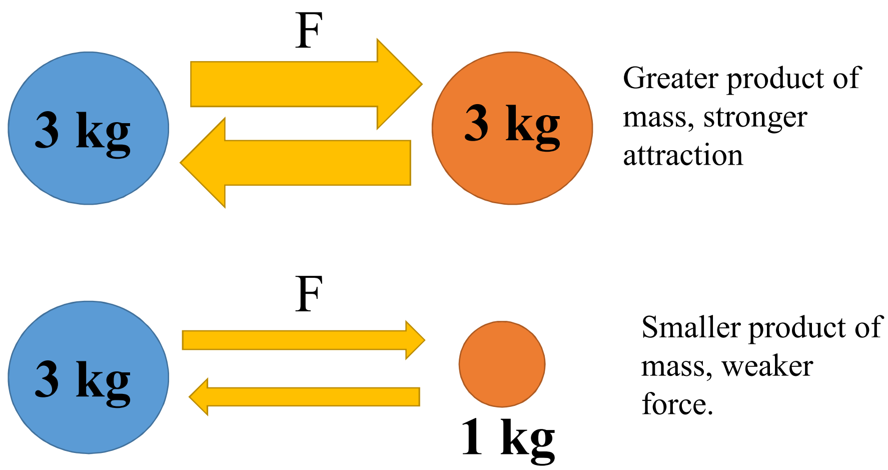
Force of sun pulling on Earth is equal in magnitude and opposite in direction to Earth pulling on the Sun.



The attraction force is equal and opposite despite the size difference.

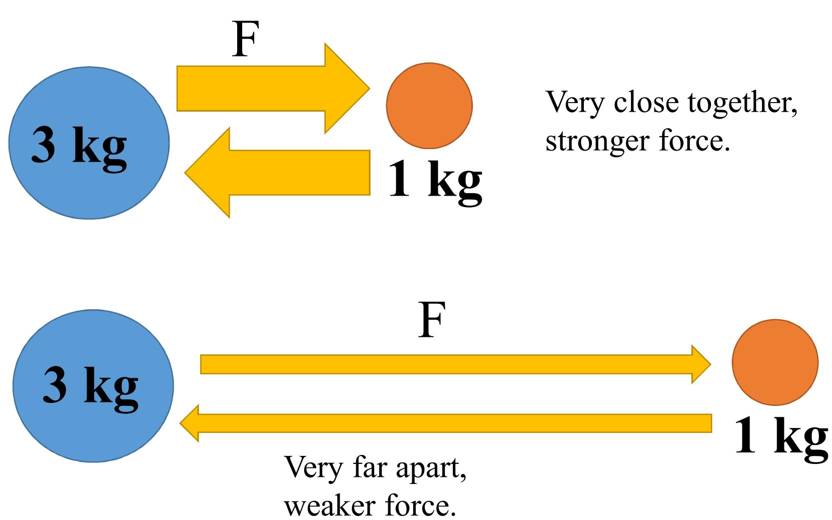
Masses of objects are multiplied in the numerator

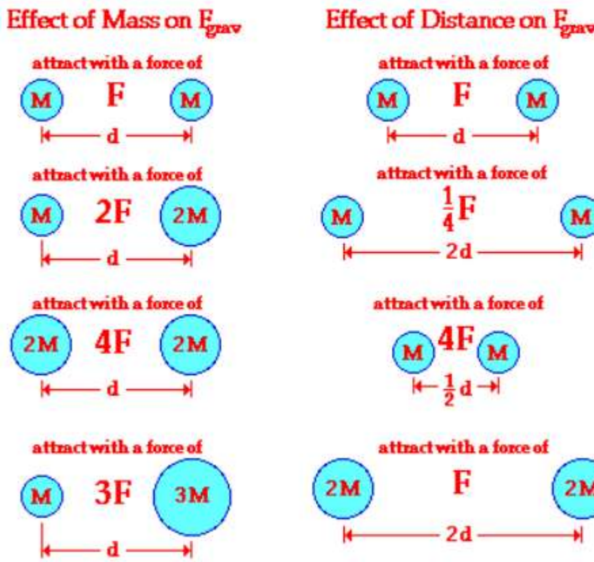
- Greater the masses, stronger the force
- Lesser the masses, weaker the force



Distance between objects is in the denominator (squared)

- Greater the distance, weaker the force
- Lesser the distance, stronger the force





Weight & Gravitational Fields

MASS

Mass NEVER changes.

Mass is CONSERVED.

The mass of an object always remains constant regardless of:

- **Location of the object (including on Earth or on another planet)**
- **Motion or movement of the object.**
- **Forces acting upon the object.**

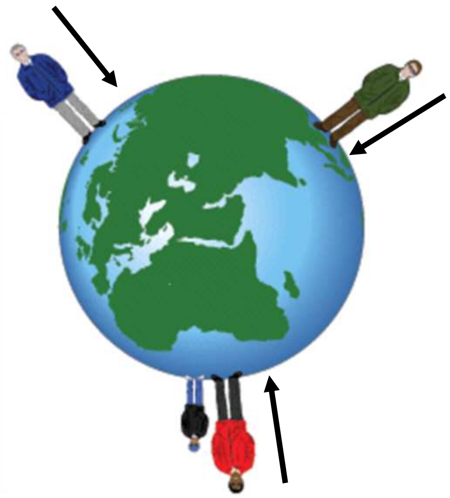
Gravitational Fields

An object with mass generates a ***gravitational field*** in the space surrounding the object.

Another object placed in the gravitational field will experience a gravitational force.

Gravity Fields on Planets

The gravity field of the planet exerts a force on objects toward the planet's center of mass. (the solid surface stops objects from moving downward)



Calculating Gravitational Field

$$g = G \cdot \frac{M}{r^2}$$

M = mass of planet (kg)

r = distance from center of planet (m)

G = universal gravitational constant

$$G = 6.67 \times 10^{-11}\, N\frac{m^2}{kg^2}$$

Calculating Gravitational Field

$$g = G \cdot \frac{M}{r^2}$$

g is the strength of the gravitational field

g is also called the ***acceleration due to gravity***

The strength of a planet's gravity field is **proportional to the mass of the planet** and **inversely proportional to distance** from the center of the planet.

- Greater mass of planet, stronger gravity field.
- Lesser mass of planet, lesser gravity field.

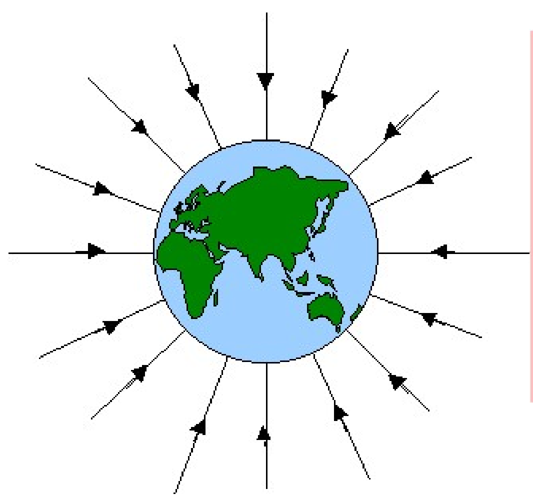
- Farther from the center, weaker gravity field.
- Closer to the center, stronger gravity field.

Calculating g at Earth's Surface

Mass of Earth: $M_{Earth} = 5.97 \times 10^{24}$ kg

Radius of Earth: $r_{Earth} = 6.37 \times 10^{6}$ m

$$g_{Earth} = (6.67 \times 10^{-1}\ )\frac{(5.97 \times 10^{24})}{(6.37 \times 10^{6})^{2}} = 9.81\ m/s^{2}$$



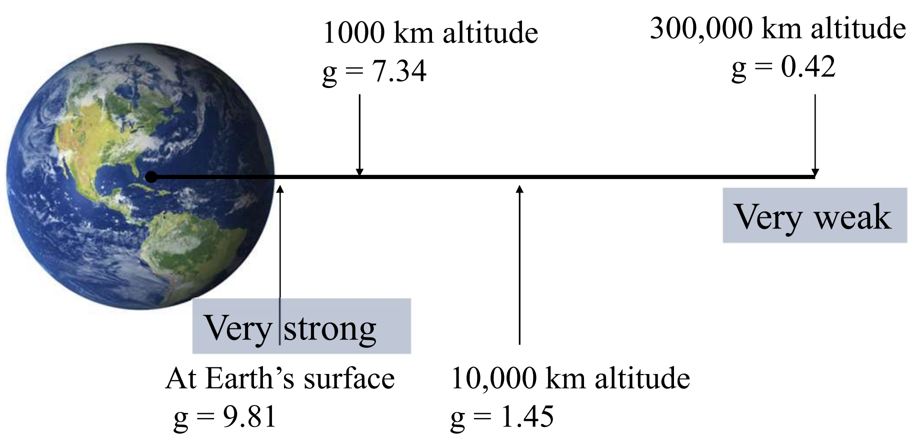
Gravity fields become progressively stronger closer to the planet.

Gravity fields become progressively weaker with increasing distance away from the object.

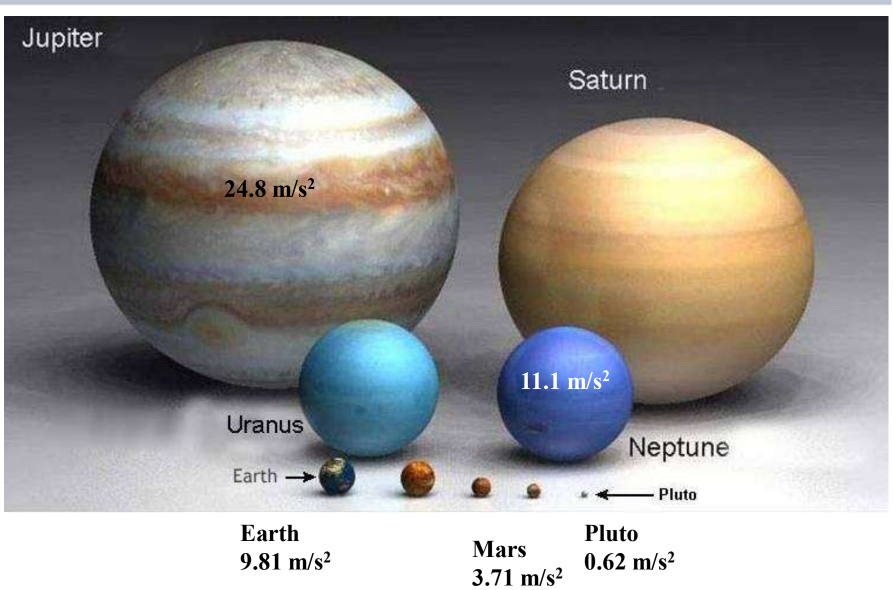
Earth's gravity field strength at Earth's surface is g = 9.81 m/s^2. The Earth accelerates objects downward at 9.81 m/s^2.

Earth's gravity field becomes weaker with increasing distance from the planet's center of mass.

- Higher altitude, lesser acceleration, weaker attraction



Bigger the mass of the planet or moon, the stronger the gravity field.



Gravity fields on other bodies in our solar system.

Planet/Moon	g
Moon	1.62 m/s^2
Mars	3.71 m/s^2
Venus	8.87 m/s^2
Earth	9.81 m/s^2
Jupiter	24.9 m/s^2

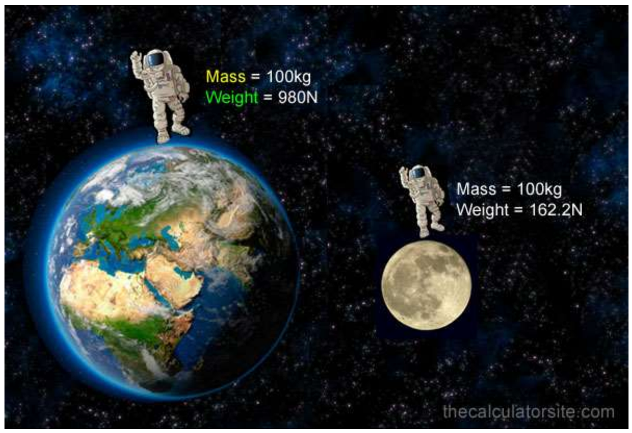
Weight

- Force pulling down on object
- Force of gravity on an object in a planet or moon's gravity field

$$F_g = m \cdot g$$

F_g = weight (N)
m = mass of object (kg)
g = gravity field/acceleration due to gravity (m/s^2)

Weight is the product of mass (m) and gravity field strength (g).
- The astronaut's mass is conserved.
- His weight is lesser on Earth's moon the moon's gravity field is weaker than Earth's gravity field.



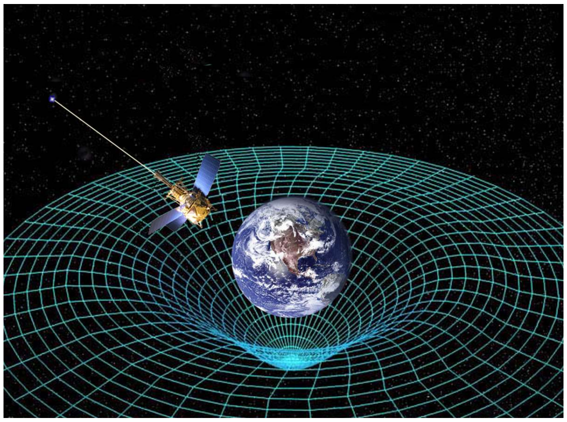
Calculating Escape Velocity

$$v_e = \sqrt{\frac{2 \cdot G \cdot M}{r}}$$

v_e = escape velocity (m/s)
M = mass of planet or moon (kg)
r = radial distance from center of mass (m)
G = Universal gravity constant

$$G = 6.67 \times 10^{-11} N \frac{m^2}{kg^2}$$

According to **Einstein's General Theory of Relativity**, gravity warps **spacetime**. The diagram shows the theoretical 3-dimensional warping of spacetime by the Earth. Gravity is strongest closest to the Earth, it becomes weaker with increasing distance.



The greater the object's mass, the stronger the object's gravity field, the greater the warping of spacetime by gravity.

Free Fall

How will a dropped or thrown object move?

Free Fall

When the *only* force being exerted on an object is the force of gravity, we say that the object is in **free fall**

In other words…

- Nothing is touching, holding, or supporting the object
- There is no air resistance

Free Fall

NOTE:

An object does NOT have to be moving downward to be in free fall

For example: a baseball thrown upward is in free fall if gravity is the only force acting on it

Acceleration Under Free Fall

ALL objects under free fall (no air resistance) will have the same downward acceleration, *regardless of their mass*

Acceleration Under Free Fall

All objects, regardless of mass, will have the following acceleration under free fall:

$$a_y = -g = -9.81 \text{ m/s}^2$$

g is a defined constant called the *acceleration due to Earth's gravity*

$$g = +9.81 \text{ m/s}^2$$

Free Fall Kinematic Equations

How fast is something moving after I drop it?

How fall has it fallen after I dropped it?

Free Fall Kinematic Equations

- Under free fall, acceleration is constant, so the kinematic equations can be used

$$v_f = v_i + at$$

$$\Delta x = v_i t + \frac{1}{2}at^2$$

- Just plug in $\boldsymbol{a = -g = -9.81\ \mathrm{m/s^2}}$

Calculating free fall velocity

$$v_f = v_i - gt$$

v_f = final velocity (m/s)
v_i = initial velocity (m/s)
g = 9.81 m/s^2
t = time of flight (s)

Initial velocity $v_i = 0$ if object is dropped from rest.
Initial velocity v_i is (+) if object initially thrown up.
Initial velocity v_i is (–) if object initially thrown down.

Calculating the vertical displacement the object falls (drop distance).

$$\Delta y = v_i t - \frac{1}{2} g t^2$$

Δy = change in height (drop distance) (m)
v_i = initial velocity (m/s)
g = 9.81 m/s^2
t = time of flight (s)

Freefall Problems

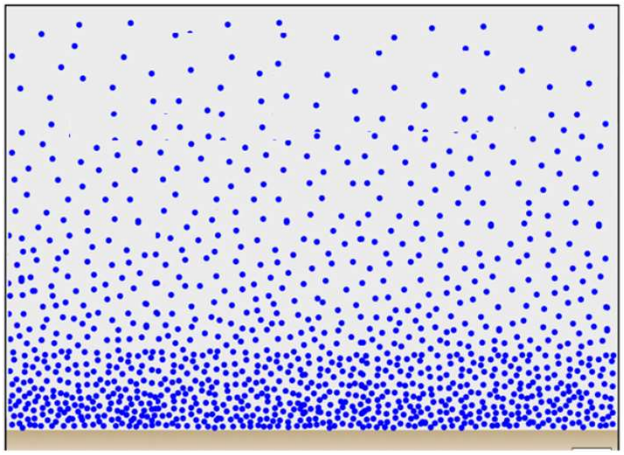
Three scenarios:

- Object dropped from rest (v_i = 0 m/s)
- Object thrown upward (v_i is positive)
- Object thrown downward (v_i is negative)

Air Resistance & Terminal Velocity

How is skydiving possible?

Earth's atmosphere is filled with air (gases).

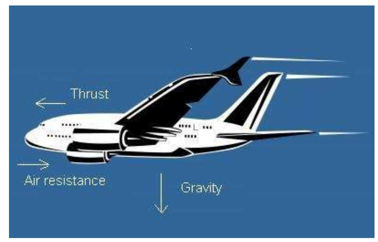


As objects move through the atmosphere, the objects will feel the effects of **air resistance force**.

What is Air Resistance Force?

- Resisting or dissipative force experienced by an object passing through air.
- Collisions with air molecules. The air molecules push back on the object at the point of collision.
- Air resistance reduces the object's kinetic energy and produces heat.
- Cause freefalling objects to achieve **Terminal velocity**

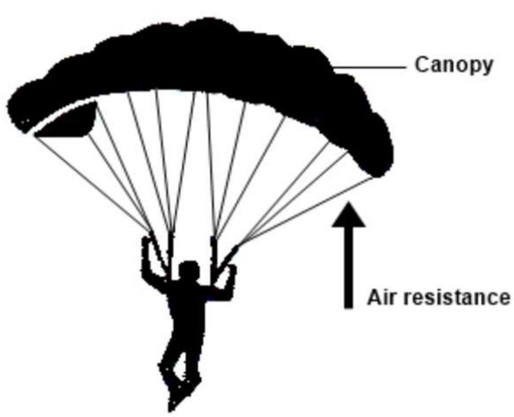
Air resistance is a "push-back" force. As the object moves through air, the collision with air molecules causes the air molecules to "push back" in the opposite direction of the object's motion.



Falling objects experience the upward air resistance

Flying objects experience air resistance in the direction opposite of flight.

Air resistance is maximized by larger surface areas. Larger surface areas collide with more air molecules, which creates a greater "push back" force by air.

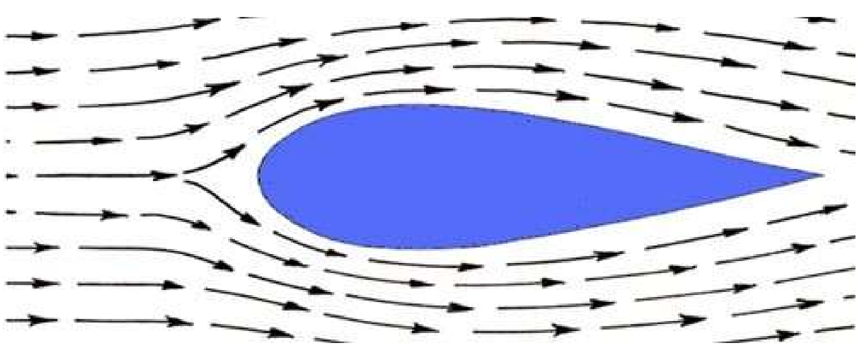


Parachute canopies are designed to create large surface areas to catch the most air molecules.

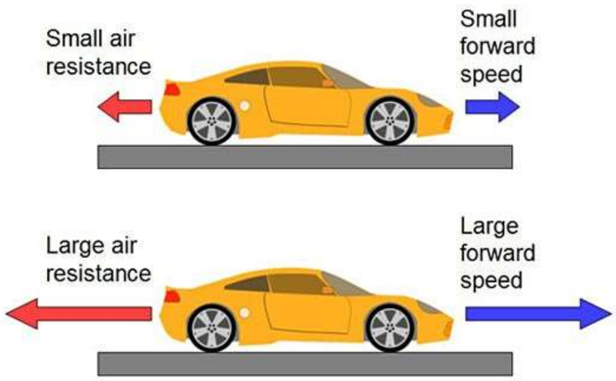
Aerodynamic contours reduce air resistance—fewer collisions with air molecules and air currents flow around the object.

Higher profile shapes with greater surface area increases air resistance—more collisions with air molecules and more air turbulence.

Teardrop: Most Aerodynamic Shape



Air resistance force is **proportional to the velocity** of the moving object. Faster = greater air resistance. Slower = lesser air resistance.

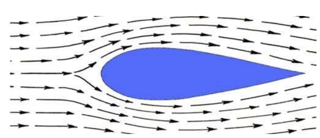


The faster the object moves, the more collisions with air molecules, the greater the air resistance force.

Air Resistance

The amount of air resistance experienced by an object is dependent upon three things:

1. Shape
   - More aerodynamic = less air resistance

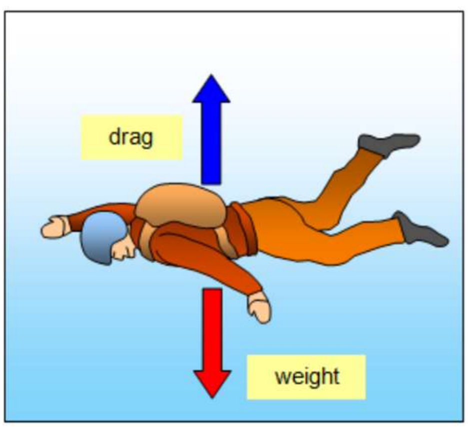


2. Surface area
   - More surface area pushes more air = more air resistance

3. Speed

Terminal Velocity

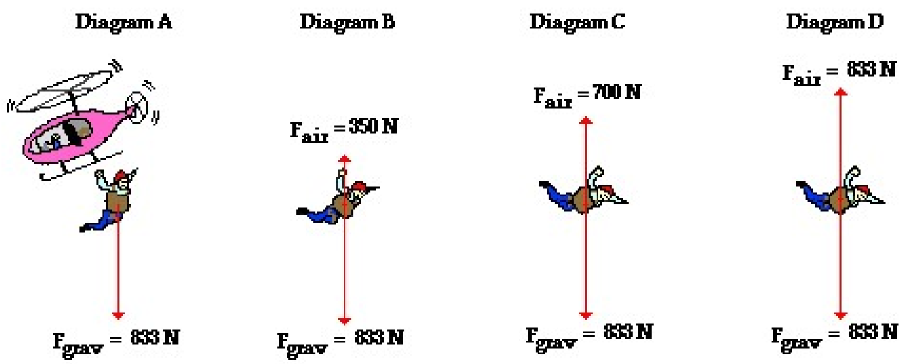
When the air resistance pushing up on a falling object balances the gravity pulling down on the object, there will be no acceleration and the object will travel at a constant velocity, called the **terminal velocity**



Falling objects eventually reach **terminal velocity** if allowed to fall for a long enough time.

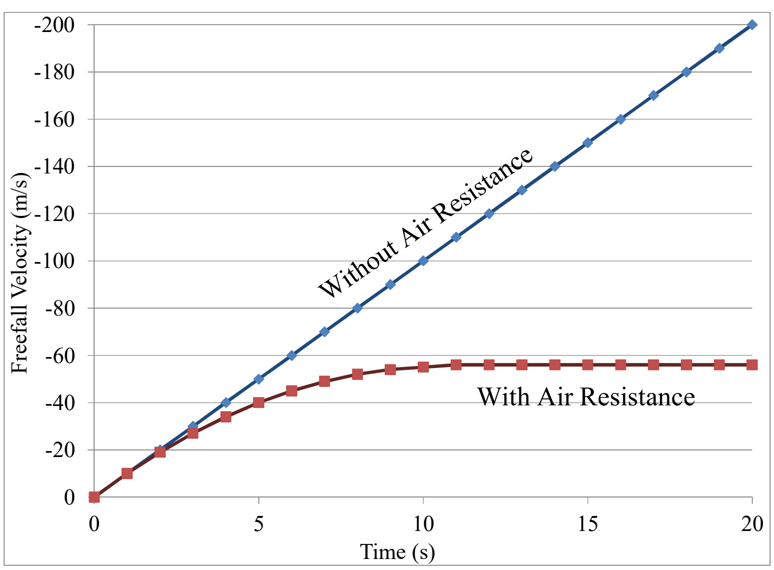
- Balanced forces. Downward gravity force (weight) is equal and opposite the upward air resistance force.
- Acceleration = 0.
- Objects fall at a constant velocity (do not speed up or slow down).

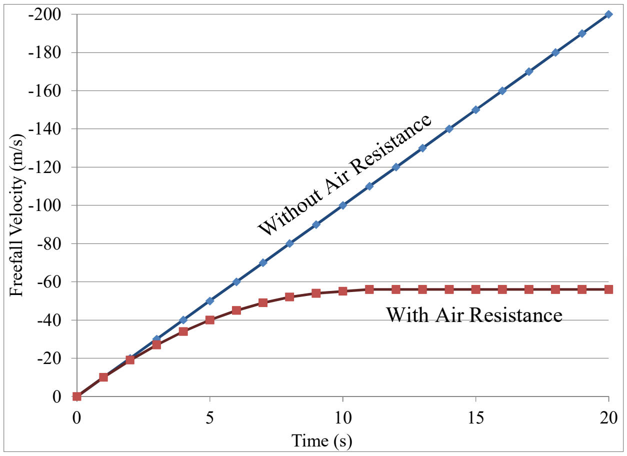
At jump | Weight > Air resistance | Terminal velocity



As freefall velocity increases, air resistance force increases until the air resistance force is equal and opposite of the object's weight (gravity force).

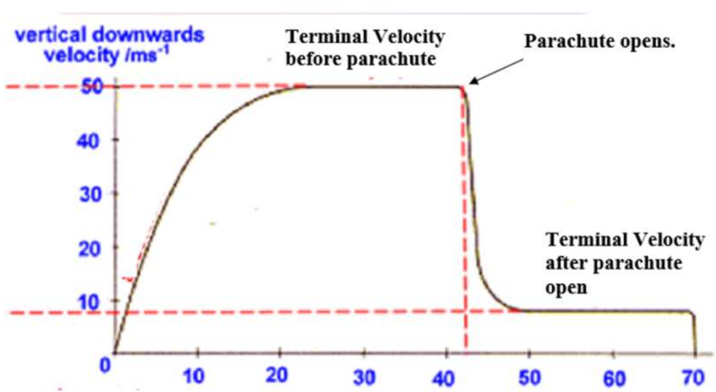
Without air resistance, objects would fall faster and faster with the same acceleration (9.8 m/s^2). (blue line)





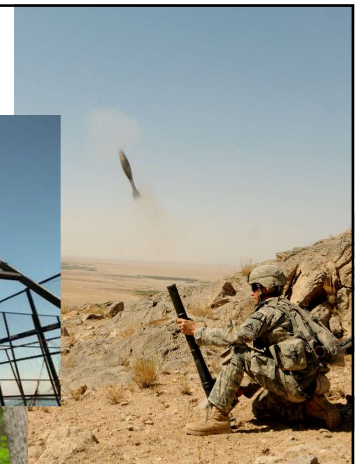
Terminal velocity

With air resistance, objects would fall faster and faster, but the acceleration (rate of velocity change) would decrease. Eventually, acceleration will reach 0 and the object will fall at a constant velocity. (red line)



A freefalling jumper will achieve a much faster terminal velocity before the parachute opens. As the parachute opens, the parachute instantly increases the surface area. That makes greater air resistance push-back force, slowing the fall instantly. The new terminal velocity after the parachute opens is much lower as the parachute drifts to the ground.

Projectile Motion



What is projectile motion?

- Similar to freefall

In projectile motion:

- Only force on the object is gravity
- Object initially launched either vertically, horizontally, or at some angle between the two

Examples: shooting a basketball, bow and arrow, a cannon

Projectile Motion

Projectile motion is two-dimensional (2D) motion

Projectile velocity is split into two components:

Horizontal (left-right) component: $\vec{v}_x$

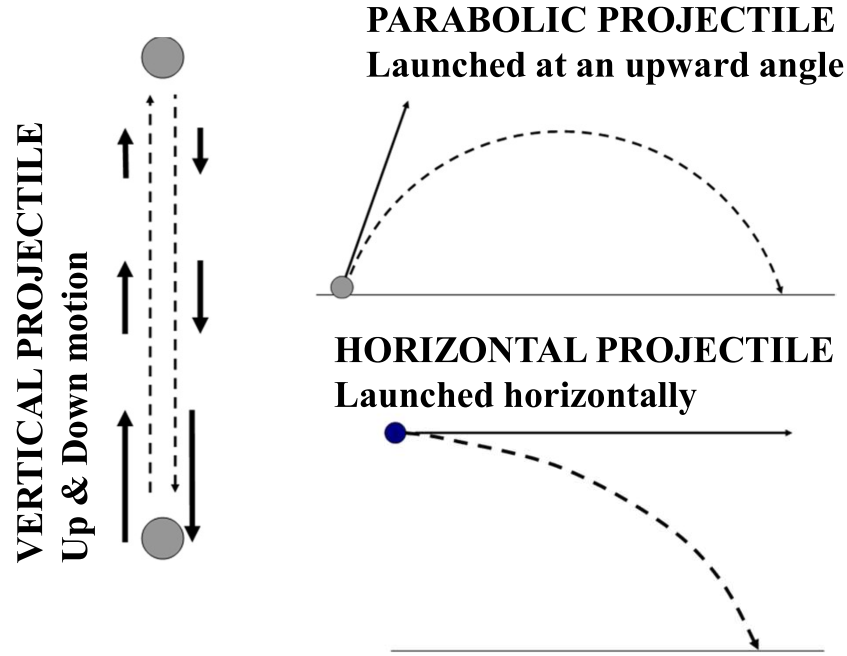
Vertical (up-down) component: $\vec{v}_y$

Golden Rule of Projectile Motion

The horizontal movement and the vertical movement of the projectile act independently of each other.

The horizontal movement follows the regular kinematic equations.

The vertical movement follows the free fall kinematic equations.



Vertical projectile motion: Projectile launched at a 90º angle (straight up). Its trajectory is up and down. There is no range, launch and impact are at the same position.

Horizontal projectile: Launched at 0º angle (horizontally). Its trajectory is a downward curve. There both has vertical and horizontal motion.

Parabolic projectile: Launched at an upward angle (1º-89º); The trajectory looks like an upside down U-shape. Has vertical and horizontal motion.

Parameters

- **Trajectory:** the path of flight through the air.

- **Range (Δx)**: the horizontal displacement; the horizontal change in position from launch to impact.

How far away did the object fly from its launch position.

- **Height (Δy):** the vertical position; how high above the ground the projectile got.

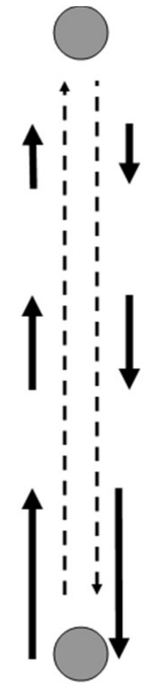
- **Time of flight (t)**: the amount of time the projectile remained airborne between launch and impact.

- **Initial launch velocity (v_o):** how fast the projectile was launched. The initial velocity of the projectile.

- **Launch angle:** the angle of the projectile's launch.
  - 90º for vertical projectiles.
  - 0º for horizontal projectiles.
  - 1º-89º for parabolic projectiles.

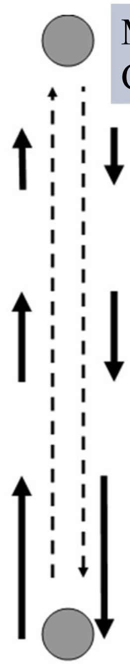
Vertical Projectile Motion

- Launch angle = 90º
- Projectile is launched straight up into the air.
- Flies upward, reaches a highest position, falls down to the ground.
- Only height changes
- Zero range (horizontal displacement)



At highest position

Motionless for an instant
Changes direction



Upward velocity decreases; gets slower as it moves upward

Downward velocity increases; gets faster as it moves downward

At launch: fastest velocity in the up direction

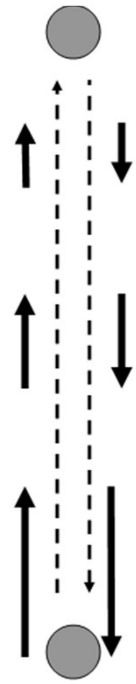
At impact: fastest velocity in the down direction

Maximum height Δ**y** above the ground depends on the initial launch velocity.

- Faster launch velocity = higher
- Slower launch velocity = lower

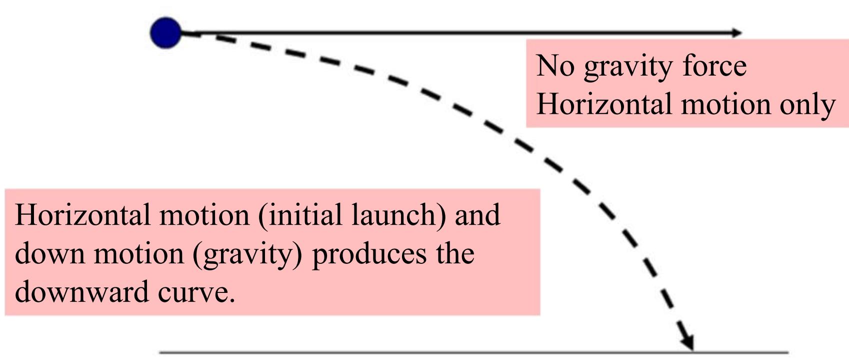
Time of flight depends on the maximum height above the ground.

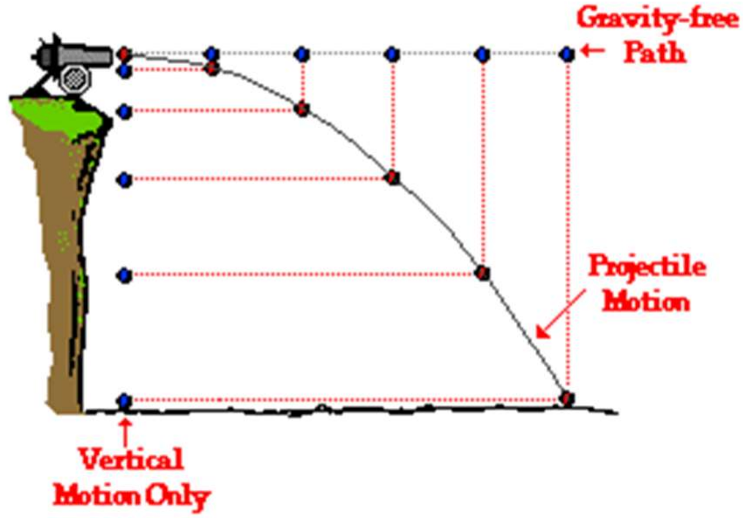
- Higher max height yields longer time of flight.
- Lower max height yields shorter time of flight.

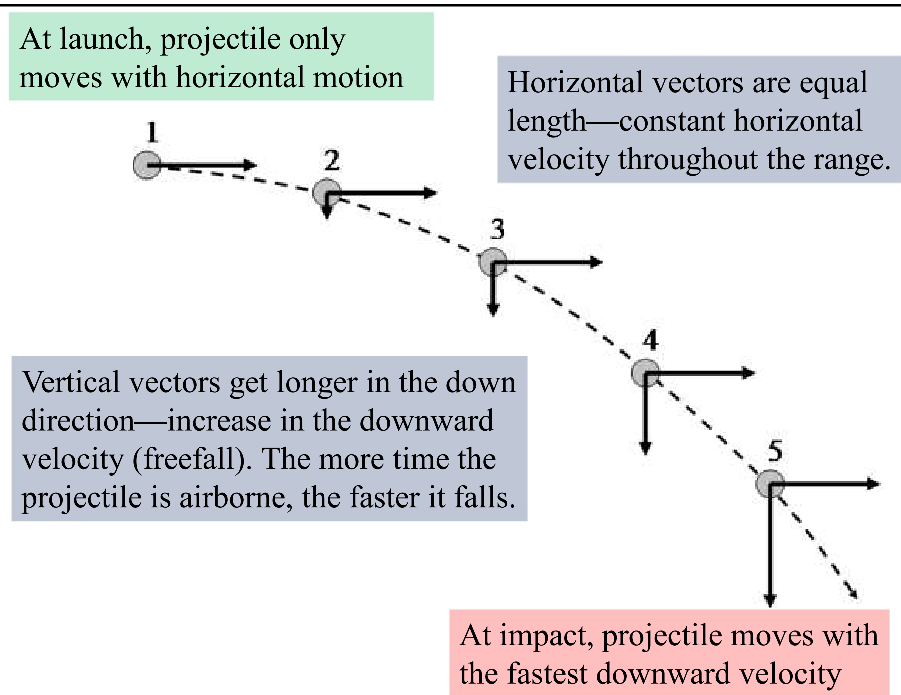


Horizontal Projectile Motion

- Object is launched with initial horizontal motion.
- Follows a downward curve trajectory because gravity pulls down on the object as it moves away from launch—*it moves away and falls at the same time*.

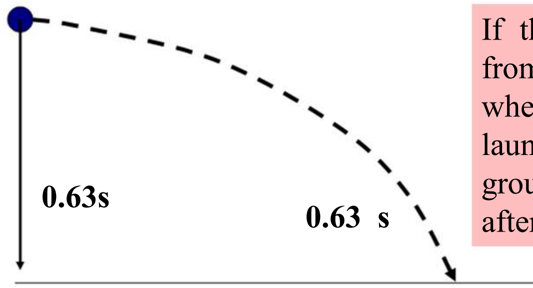



Gravity-free
Path
Projectile
Motion
Vertical
Motion Only

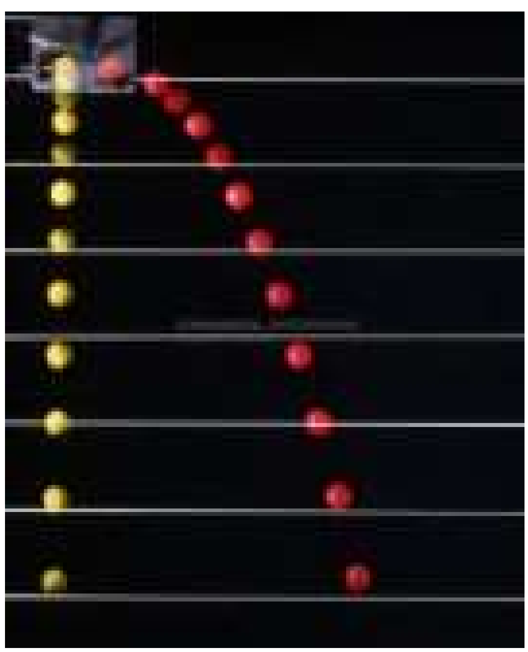

At launch, projectile only moves with horizontal motion
Horizontal vectors are equal length—constant horizontal velocity throughout the range.
1
2
3
4
5
Vertical vectors get longer in the down direction—increase in the downward velocity (freefall). The more time the projectile is airborne, the faster it falls.
At impact, projectile moves with the fastest downward velocity

Time of flight only depends on the launch height above the ground.

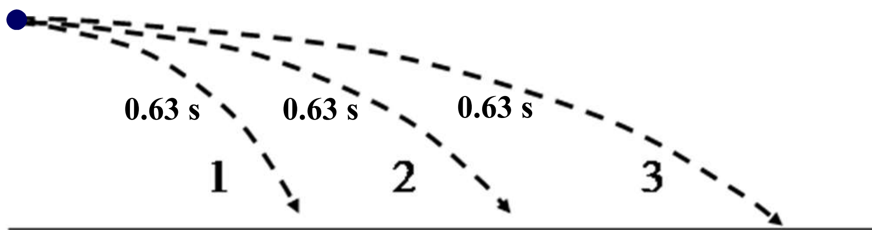
- Higher the launch position, the longer the time of flight.
- Time of flight is independent on initial horizontal launch velocity or range.



If the projectile was dropped from the same exact position at where it was horizontally launched, it would impact the ground with the same time after release.



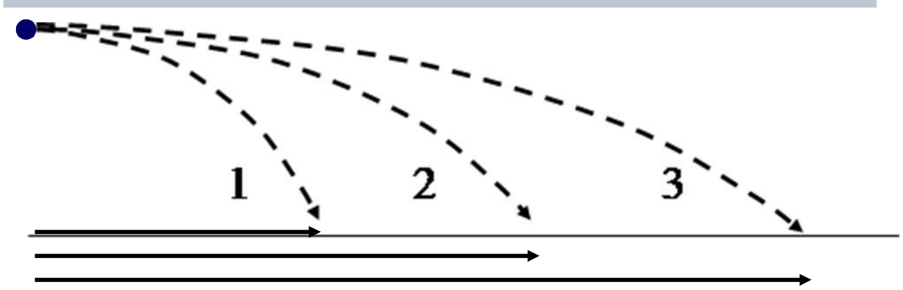
If multiple horizontal projectiles were launched from the same height above the ground but with different initial horizontal velocities, all will have the same time of flight but differ in the ranges.



All will impact the ground with the same time of flight because they have identical launch heights—they fall the same downward distance.

The range **Δx** of the projectile is controlled by the initial launch velocity.

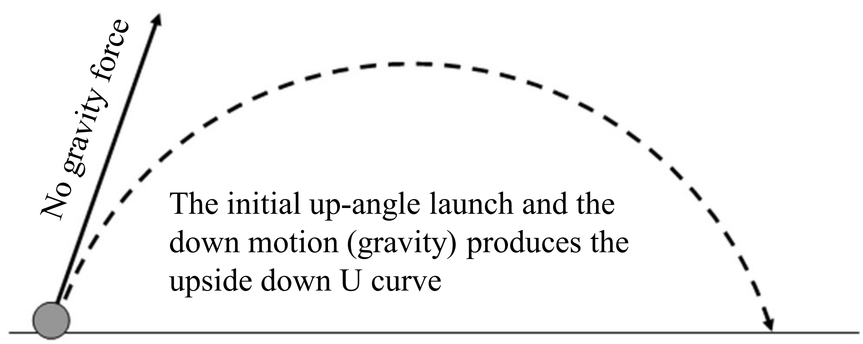
- Faster launch velocity yields longer range.
- Slower launch velocity yields shorter range.



Projectile 1 had the slowest launch velocity, therefore it will have the shortest range. Projectile 3 had the fastest launch velocity, therefore it will have the longest range.

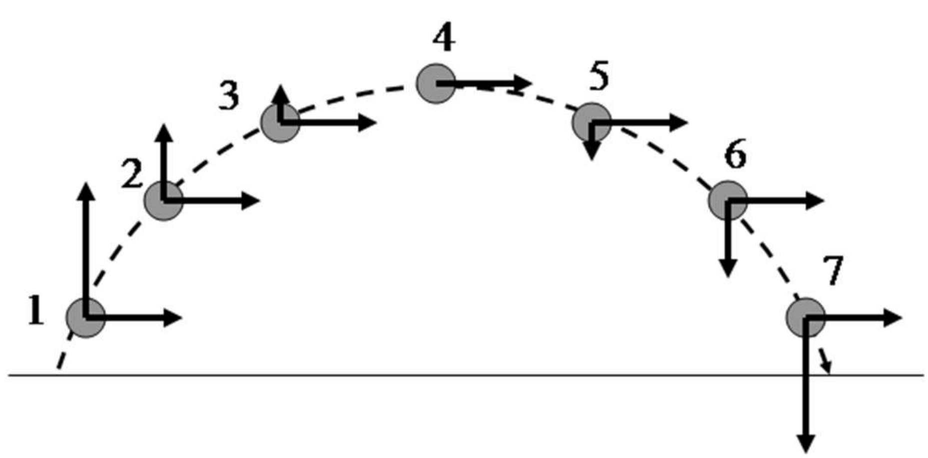
Parabolic Projectile Motion

- Object is launched with a diagonal launch angle (1-89º).
- Follows a parabola shape trajectory—initially moves upward but gravity eventually causes the projectile to move downward



Horizontal velocity vectors are equal length. There is constant horizontal velocity throughout the range.

- The object moves away horizontally from launch at the same rate as it moves through the air.

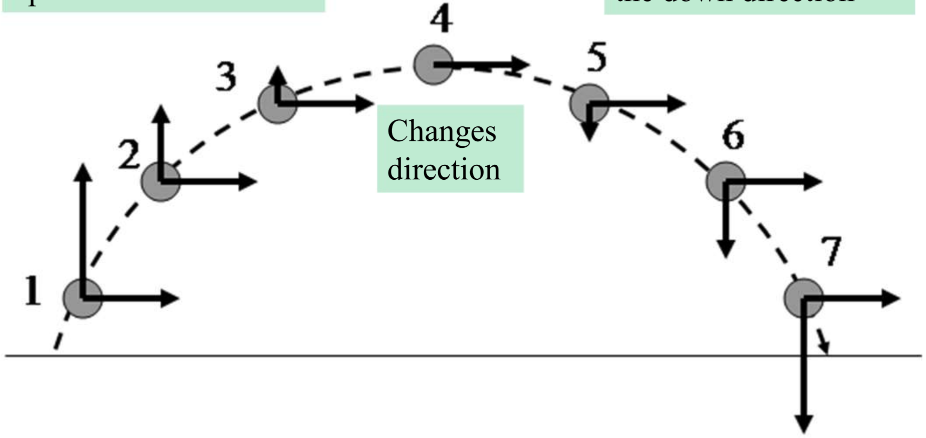


Vertical velocity vectors change length and change direction in flight.

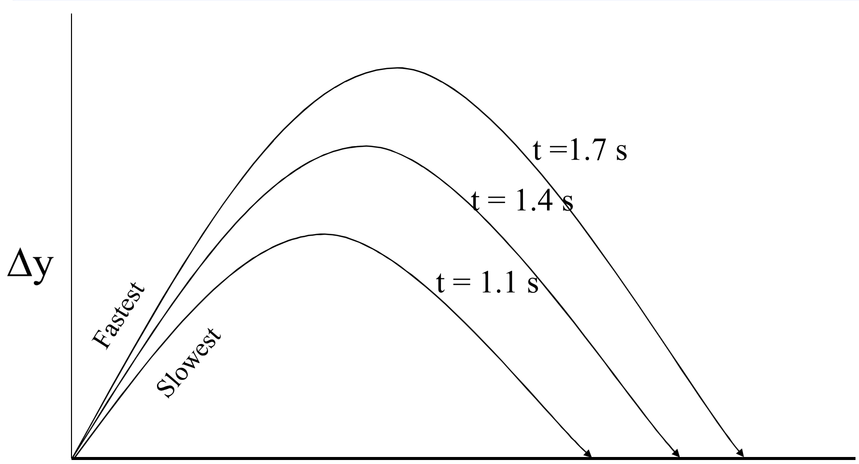
Vertical vectors get shorter: slows in the up direction

Horizontal only at the max height

Vertical vectors get longer: gets faster in the down direction



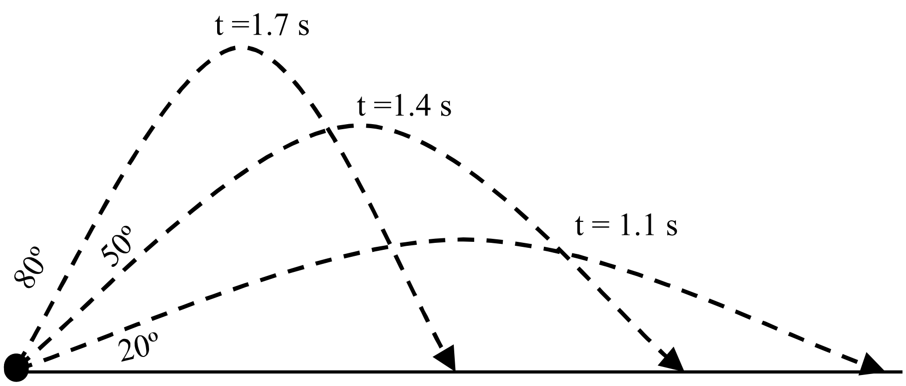
The greater the initial launch velocity, the higher the maximum height above the ground, the longer the time of flight.



The three projectiles have the same launch angle, but different initial launch velocity.

The greater the initial launch angle, the higher the maximum height above the ground, the longer the time of flight.

The three projectiles have the same launch velocity, but different launch angles. The projectile that fires more upward will produce a higher maximum height and a longer time of flight.



Uniform Circular Motion

How can an object accelerate?

1. Speed increases
2. Speed decreases
3. **Direction changes**
4. Change of speed and direction

Uniform Circular Motion

An object is in uniform circular motion when these two conditions are true:

- The path of motion is along a circle
- The speed is constant

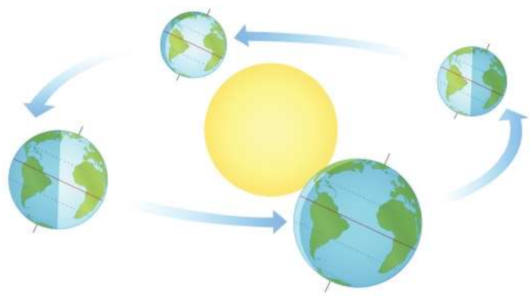
In uniform circular motion, magnitude of velocity remains constant but direction of velocity is always changing

Since velocity is always changing, uniform circular motion is a state of acceleration!

Two forms of circular motion:

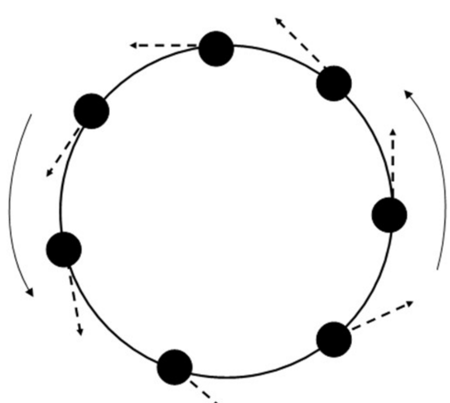
- **Rotation: the spinning motion around the center of mass**
- **Revolution: the movement of one object in a circular path around another object (orbiting)**

Rotation
The Earth **rotates** (spins on its axis) 1 rotation every 24 hours (1 day)



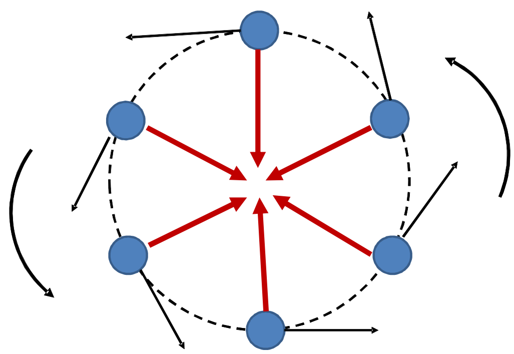
Revolution
The Earth **revolves** around the sun in its orbit 1 revolution every 365.34 days (1 year).

As an object rotates or revolves, the **instantaneous velocity is constantly changing** because direction is constantly changing. Therefore, rotating or revolving objects are **constantly accelerating** as they move.



The dashed vector arrows show the **tangential velocity** (the instantaneous velocity) of the object moving in circular motion.

Tangential velocity (solid black arrows): The instantaneous velocity of the moving object

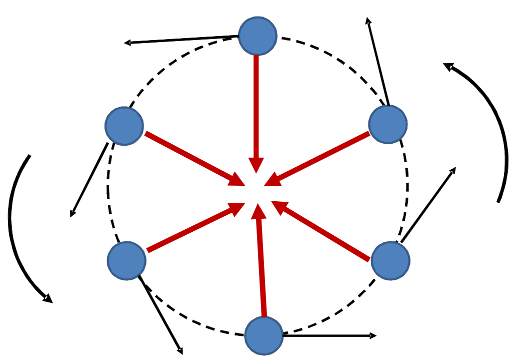


On its own, the object will move in a straight line tangential to the circular path because of its inertia.

A "center seeking" force (like tension or gravity) pulls inward.

Circular motion is established when the inertia is balanced by the "center seeking" force.

Centripetal Acceleration is the change in velocity per time experienced by a rotating/revolving body.

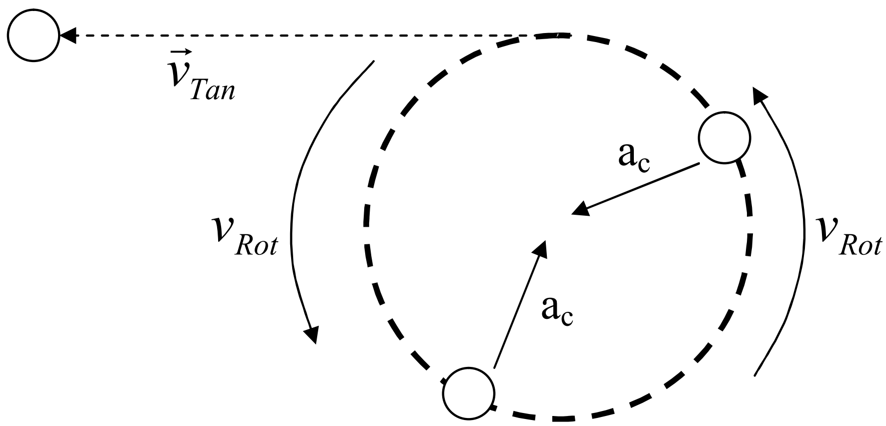


Centripetal acceleration points inward toward the center of the circle.

(red vector arrows)

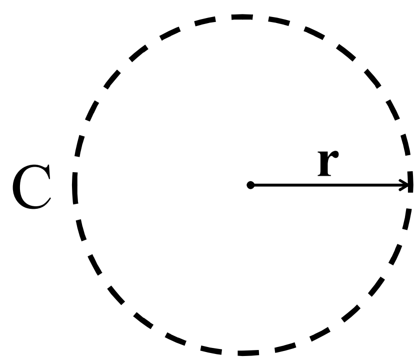
Centripetal acceleration results from a "center seeking" force.

Remember: An acceleration is defined as the change in velocity with time. If direction changes, the velocity changes and an acceleration has occurred.



The object wants to keep move in a straight line because of ***inertia***. If the object is released, it will move in straight line with a ***tangential velocity*** equal to the ***rotation speed***.

- **RADIUS (r)**
- **DIAMETER (D)**
- **CIRCUMFERENCE (C)**

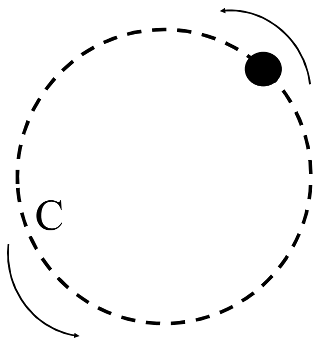


$$C = 2 \cdot \pi \cdot r$$

$$\pi \approx 3.1416$$

Circumference defines the linear distance around the perimeter of the circle.

Linear rotation speed (Revolution speed)



$$v = \frac{n \cdot C}{t}$$

v = linear rotation speed (m/s)
n = number of spins in time t
C = circumference (m)
t = time (s)

Linear rotation speed defines how fast the object moves in a circular motion—distance per second.

Angular Velocity

- How fast an object moves in a circular path based on how much angle turned.
- Radians of curvature per second.

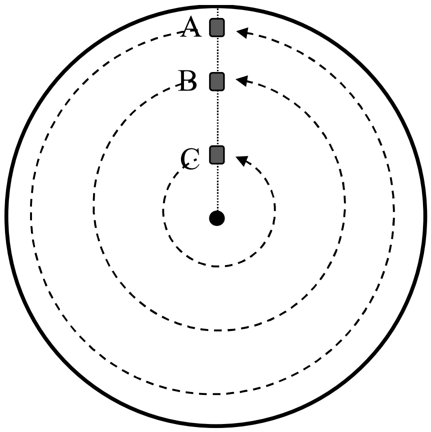
Radian = the arc-length equal to 1-radius length.

Circumference = 360° = 2π radians

$$\omega = n\frac{2\pi}{t}$$

ω = angular velocity (rad/s)
n = number of rotations
t = time (s)

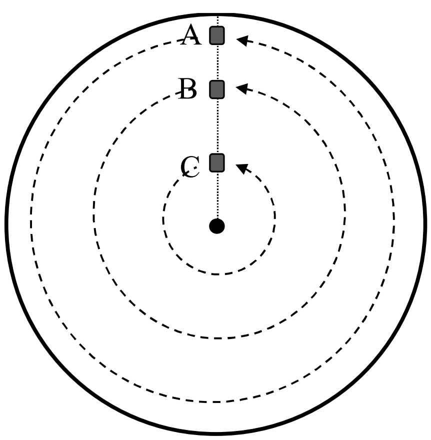
Let the disk rotate 1 time in 10 seconds



Points A, B, and C will have **equal angular velocities** because they all complete the same number of rotations (1 circle, 360º) in the same amount of time (10 sec).

$$\omega = \frac{2\pi}{10s} = 0.628\ {}^{rad}/_{s}$$

Let the disk rotate 1 time in 10 seconds



Points A, B, and C will have **different linear rotation speeds** because the radii are different.

"A" rotates the fastest (largest circumference, travels greatest distance per time)

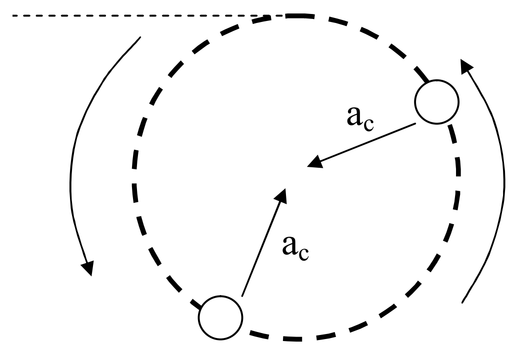
"C" rotates the slowest (smallest circumference, travels least distance per time)

$$v = n \cdot \frac{C}{t} = n \cdot \frac{2\pi \cdot r}{t}$$

Centripetal Acceleration

$$\boxed{a_c = \frac{v^2}{r}}$$

a_c = centripetal acceleration (m/s²)
v = linear rotation speed (m/s)
r = radius (m)



Centripetal acceleration is the "center seeking" acceleration. It defines how fast the object changes velocity by constantly changing direction.

Centripetal Force

Centripetal acceleration is caused by centripetal force

Centripetal force:
- A "center-seeking" force that causes circular motion
- Always points toward the center of the circle
- NOT a new kind of force
  - Can be normal force, friction, gravity, tension

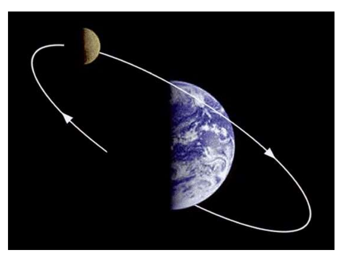
Examples of Centripetal Force

Normal force – Gravitron ride

Friction – car on a curve

Examples of Centripetal Force

- Gravity – Moon orbiting the Earth



- Tension – swinging object attached to a string

Newton's 2nd Law for Circular Motion

$$F_c = m \cdot a_c$$

$$= \frac{m \cdot v^2}{r}$$

F_c = centripetal force (N)
m = mass (kg)
a_c = centripetal acceleration (m/s^2)

Angular Momentum

- The intensity of rotational motion.
- **Product of the mass of the rotating object (m), rotation speed (v), and turn radius (r).**

$$L = r^2 \cdot m \cdot \omega = r \cdot m \cdot v$$

m = mass
ω = angular velocity (rad/s)
n = number of rotations
r = turn radius (m)
v = rotation speed (m/s)

Law of Conservation of Angular Momentum

In the absence of an external torque (rotation force), the angular momentum of an object in a closed system will remain constant.

- The rotation speed of the spinning object is inversely proportional to the turn radius of the rotating mass.
- The product of the **radius** multiplied by the **linear rotation speed** must always remain constant.

Changes in r and C create changes in v

- If **r (radius)** gets smaller, **v (linear rotation speed)** has to increase accordingly to compensate for the smaller radius—the object spins faster, more turns per second

- If **r (radius)** gets larger, **v (linear rotation speed)** decreases accordingly to compensate for the greater radius—the object spins slower, fewer turns per second.

$$L = r \cdot m \cdot v$$

$$L = r \cdot m \cdot v$$

Larger turn radius (arms extended), slower rotation speed (v goes down).

Smaller turn radius (arms tucked), faster rotation speed (v goes up).

Circular Motion Problems

Identify all of the parameters in the calculation. Write them. Make sure time and radius are in the correct units.

Solve for circular motion in order

1. **CIRCUMFERENCE**
2. **ROTATION SPEED**
3. **ANGULAR VELOCITY**
4. **CENTRIPETAL ACCELERATION**
5. **CENTRIPETAL FORCE**
6. **ANGULAR MOMENTUM**

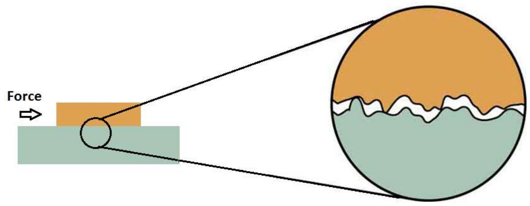
Friction

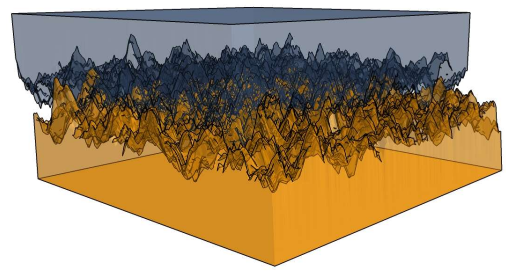
Friction ($\vec{F}_f$)

When two surfaces are in contact, **friction** force acts to oppose the movement or potential movement of the surfaces sliding over each other

- Acts in opposite direction of the movement or potential movement
- Two types:
  - Static friction: when the two surfaces are not moving relative to each other
    - Example: A box rests on a ramp. The static friction keeps the box from sliding down the ramp
  - Kinetic friction: when the two surfaces are sliding over each other

Why is there friction?





SURFACE TEXTURES

Friction depends on the combination of the object and the surface over which it moves.

- Shoes (rubber soles) have more friction against the hardwood floors.
- Socks (cotton cloth) have less friction against the hardwood floors.

Friction Coefficient

The friction coefficient (μ) designates the strength of friction between two surfaces

- The higher the number, the more friction there is

Rougher surfaces have more friction than smoother surfaces

Examples:

- Ice on ice: $\mu=0.02$
- Rubber on concrete: $\mu=1.0$

Static vs. Kinetic Friction

Kinetic friction
- Present when the two objects slide across each other
- Has one magnitude
  - Whenever the objects are sliding, friction force is constant

Static friction
- Present when the two objects are not sliding but have the potential to slide
- Has varying magnitude

Max static friction is always greater than the kinetic friction

Static vs. Kinetic Friction

Kinetic friction:

$$F_{f,k} = \mu_k F_N$$

Static friction:

$$F_{f,s} \leq \mu_s F_N$$

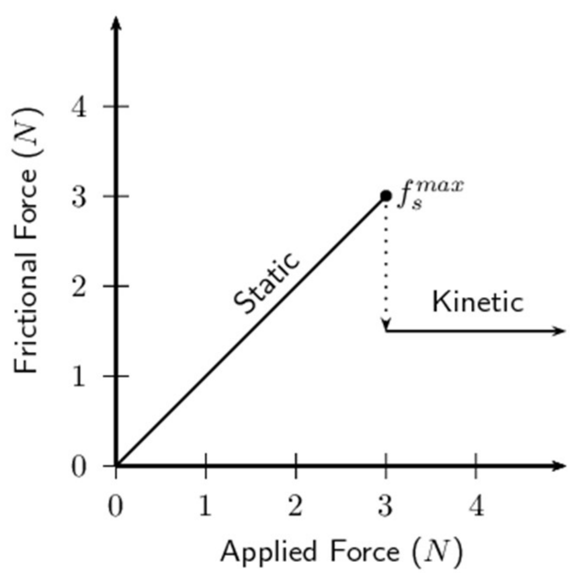
Static vs. Kinetic Friction

Maximum static friction is larger than kinetic friction

(It is harder to get an object moving than to keep it moving)

Therefore,

$$\mu_s > \mu_k$$



Friction & Normal Force

$$F_{f,k} = \mu_k F_N$$

$$F_{f,s} \leq \mu_s F_N$$

Friction depends on the **normal force** (not on the weight!)