PHYSICS: Lesson 03
CHANGES IN POSITION

Learning targets

1. Students explain and understand the difference between distance and displacement as related to changes in position and vector/scalar quantities (SP1.c)
2. Students can compare and contrast the relative changes in position between two objects from figures, experimental data, graphs, and word problems. (SP1.b, SP1.c)
3. Students can calculate distance and displacement by vector addition when given paths of travel and direction (SP1.d)

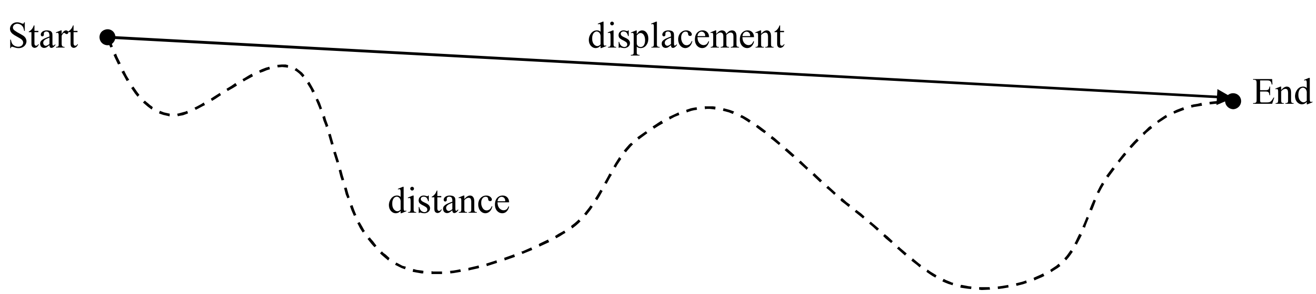
Comparing Distance and Displacement

Distance and displacement are variables that describe ***changes in position*** or "how far" an object moved. They are not interchangeable.

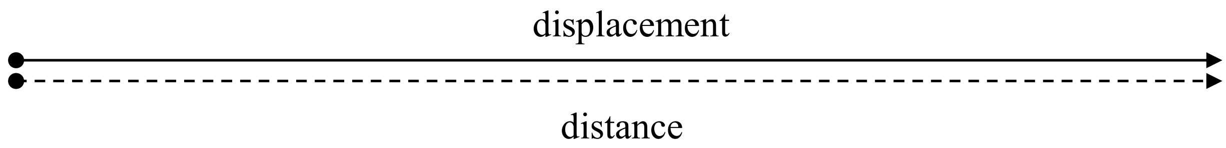
Distance is defined as the *total path length* of travel from the starting position to the end position. The symbol **d** is used to note distance. In other words, distance describes how far an object moved regardless of direction. Distance may be in a straight line (if the object in motion is moving in a straight line) or it may be in an irregular path. Distance is a ***scalar*** and only describes the total path length of travel; no direction is reported, therefore distance is always a positive number.

Displacement is defined as the *straight-line*, *absolute change in position* between the starting position and the end position. The symbol **Δx** is used to note displacement. Displacement is a vector and must have a direction (N, S, E, W, up, down, left, right). *Displacement describes the absolute difference between where motion starts and where motion ends regardless of how the object gets from the start to the finish*. Displacement may be positive or negative depending on the direction.

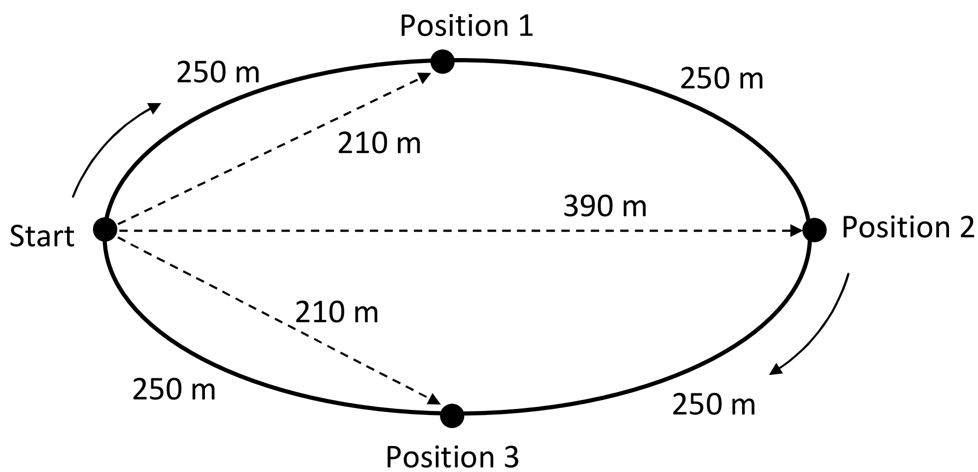
All paths of travel *have both distance and displacement* regardless if the motion is in a straight line or in a curved path. This is true because every path of travel has a starting position, and ending position, and an overall change of position.



A man walks the dashed line to move from the starting position to the end position. The dashed line is his distance of travel. The straight line represents his displacement, the absolute change in position from where he started his walk and where he ended his walk. Notice that the distance he walked is longer than the displacement. This makes sense because in real life, people and cars have to follow roads over uneven surfaces that are not straight.



In the diagram above, distance and displacement are equal in magnitude. In this case, they are equal because motion is in a straight line in one direction, the absolute difference in position from where motion started and where motion ended was the same.



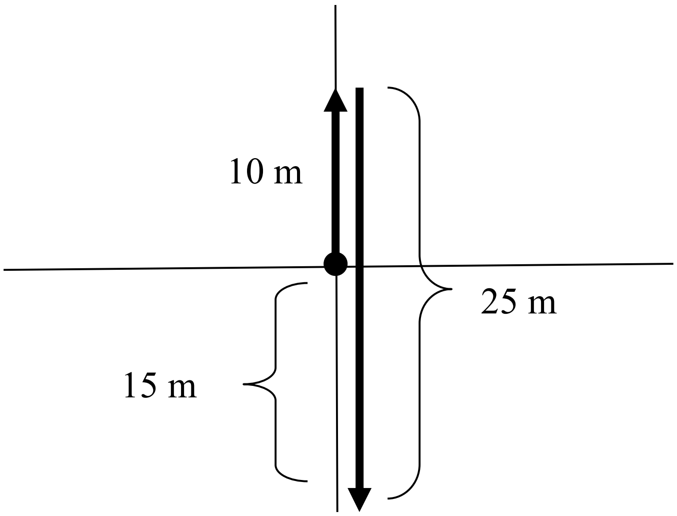
Consider a car moving around an oval race track (see picture above). A car will move around the perimeter of the race track (on the road) with distance. The straight dashed lines that connect the start to the other positions around the track represent the displacement of the car as it moves around the track. Remember, displacement is the straight-line absolute difference between the starting position and the end position of motion.

Start to Position 1	Distance = 250 meters	Displacement = 210 meters @ NE
Start to Position 2	Distance = 500 meters	Displacement = 390 meters @ E
Start to Position 3	Distance = 750 meters	Displacement = 210 meters @ SE
Start to Position 4	Distance = 1000 meters	Displacement = 0 meters

One complete circuit around the oval is 1000 meters distance. The displacement, however, for one complete circuit around the oval is zero because at start, the car's change in position between where it began its motion and where it ended its motion is zero. (Start minus Start = 0)

Calculations of Distance (d) and Displacement (Δx) Using Vectors

Suppose an object's motion is in two directions. The distance is the total length traveled (add the individual distances together). The displacement is the straight-line difference between the starting position and the ending position.



A man walked 10 m N, turned, and walked 25 m S.

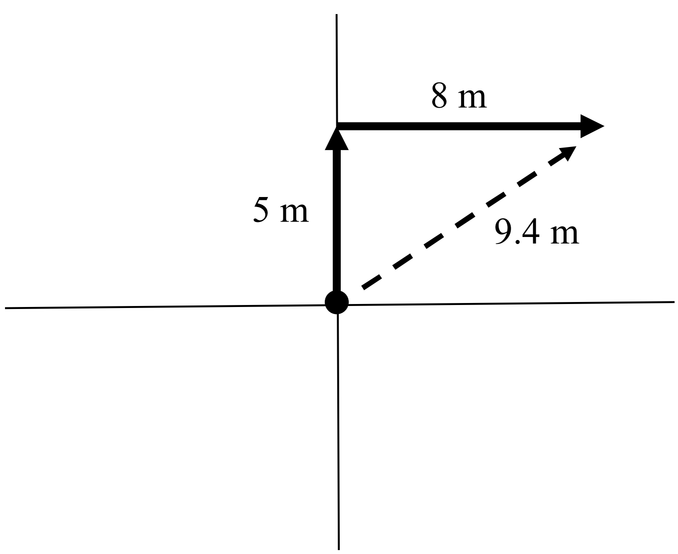
The solid arrows show how the man walked (using coordinate lines for reference.)

The man walked a total distance of 35 meters.

$$d = 10\text{ m} + 25\text{ m} = 35\text{ m}$$

The man's displacement from start to finish was Δx = 15 m S (or -15 m). (The absolute difference between starting position and ending position).

A man walked 5 meters north, turned, and walked 8 meters east. The solid arrows show how the man walked (using geographic grid lines for reference.)



The man walked a total distance of 13 meters.

$$d = 5\text{ m} + 8\text{ m} = 13\text{ m}$$

The man's displacement from start to finish was Δx = 9.4 m NE. The Pythagorean Theorem was used because the man's path of travel was in 2-dimensions where the displacement is the hypotenuse of the right triangle.

$$C^2 = A^2 + B^2$$

$$C^2 = (5\text{ m})^2 + (8\text{ m})^2$$

$$C^2 = 89\text{ m}^2$$

$$C = \sqrt{89\text{ m}^2} = 9.4\text{ m NE}$$

PHYSICS: Lesson 04
SPEED AND VELOCITY: "HOW FAST"

Learning targets

1. Students will calculate speed and velocity when given the variables distance or displacement and time. (SP1.a)
2. Students can compare and contrast the relative rates of motion (speed or velocity) between two objects from figures, experimental data, graphs, and word problems. (SP1.a; SP1.b)
3. Students can identify identical and different instantaneous speeds and instantaneous velocities by analyzing magnitude and direction of vectors.(SP1.c)
4. Students can calculate average speed, average velocity, and resultant velocity when given path of travel and total time of travel. (SP1.d)
5. Students can calculate average speed, average velocity, and describe changes in position by using motion graphs of position vs. time and velocity vs. time. (SP1.b)

Speed and velocity are ***rates***, they describe a change in position per unit time—in other words, "how fast" objects move. They are, however, calculated differently because speed is a scalar and velocity is a vector.

Speed	Velocity
• Scalar parameter • Magnitude only (how fast) • The rate of motion. • Change in distance traveled per unit time	• Vector parameter • Magnitude and direction (how fast, direction) • The rate of motion in a straight-line • Displacement per unit time

Speed is defined as how fast an object moves, ***the change in distance traveled per unit time***. In other words, speed characterizes how far you travel divided by the time it takes you to travel. As you recall, distance is a scalar and time is a scalar, therefore speed is also a scalar.

$$v = \frac{d}{t}$$

v = speed (m/s; km/hr; km/min)
d = distance (m, km)
t = time (s; min; hr)

Speed may be reported in several combinations of units: m/s, km/hr, or km/min for example.

Velocity is defined as ***the displacement or absolute change in position per unit time***. Velocity describes how fast an object moves in a straight-line per unit time at a given direction. Velocity, a vector, must always have a magnitude (how fast) and a direction (where it moves). If the rate of an object's motion changes or the direction of motion changes, velocity changes.

$$\vec{v} = \frac{\Delta x}{t} @ direction$$

v = velocity (m/s)
Δx = displacement (m)
t = time (s)

By convention, velocity is always reported in units of m/s unless otherwise noted.

Using algebra, the three-variable equation used to solve for velocity or speed may be rearranged to solve for distance, displacement, or time.

Speed or velocity	Distance or displacement	Time of travel
$v = \frac{d}{t}$ $\vec{v} = \frac{\Delta x}{t}$	$d = v \cdot t$ $\Delta x = \vec{v} \cdot t$	$t = \frac{d}{v}$ $t = \frac{\Delta x}{\vec{v}}$

Illustrative Examples

A man runs a distance of 30 meters in 4 seconds. Calculate the speed of the man. Report in m/s.

$$v = \frac{d}{t} = \frac{30m}{4s} = 7.5\,{}^{m}\!/_{s}$$

$$\xleftarrow[\text{4 s}]{\text{30 m}}$$

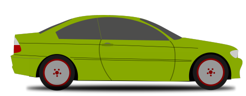
On the freeway, a car travels 500 m north in 25 seconds. Calculate the velocity of the car. Report in m/s.

$$\vec{v} = \frac{\Delta x}{t} = \frac{500m}{25s} = 20\,{}^{m}\!/_{s}$$

$$\xleftarrow[\text{25 s}]{\text{500 m}}$$

On the freeway, a car moves at a constant rate of 25 m/s for 500 m. Calculate the time traveled.

$$t = \frac{d}{v} = \frac{500m}{25\,{}^{m}\!/_{s}} = 20\,s$$



$$\xrightarrow[\text{25 m/s}]{\text{500 m}}$$

A horse ran with a velocity of 14 m/s for 6 seconds. Calculate how far the horse ran.

$$\Delta x = \vec{v} \cdot t$$

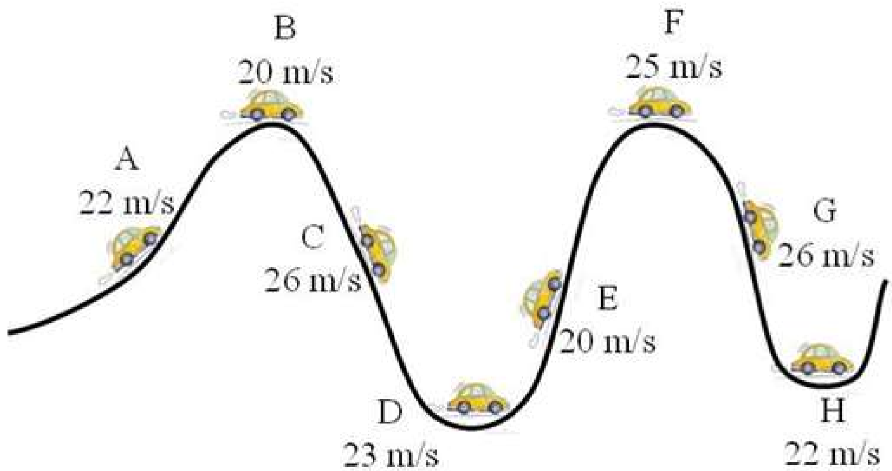
$$\Delta x = 14\,{}^{m}\!/_{s} \cdot 6\,s = 84\,m$$

$$\xrightarrow[\text{14 m/s}]{\text{6 s}}$$

Instantaneous Speed and Instantaneous Velocity

Instantaneous speed is how fast an object moves at any given second or moment in time. For example, as you ride in a car, the car probably does not maintain the same speed and direction throughout its trip. When you look at the car's speedometer—the dial or digital display that tells you how fast the car is moving—you observe the car's instantaneous speed. ***Instantaneous velocity*** is how fast an object moves in a straight-line direction at any given second in time.

The diagram below shows a car moving on a road up and down a series of hills and valleys. The magnitude of the car's instantaneous speed is provided at positions A through H on the diagram.



When the car is at positions A and H, the car is moving with the same instantaneous speed of 22 m/s. Likewise, when the car is at positions B and E, the car is moving with the same instantaneous speed of 20 m/s. For positions A and H, and for B and E, the magnitudes are the same, however, directions are different, thus they cannot have equal instantaneous velocities. Conversely, when the car is at positions C and G, the car is moving with the same instantaneous velocity of 26 m/s. They are equal instantaneous velocities because magnitude ("how fast") and direction are the same.

Average Speed and Average Velocity

Average speed and average velocity are useful to characterize the average rates of motion when objects travel long distances and do not maintain the same speed or velocity throughout the trip. Consider a school bus carrying children to school. The school bus travels a long distance, but must stop at corners and traffic signals, speed up and slow down, and travel on different roads with different speed limits and different directions as it collects children and carries the children to school. Because of those factors, the school bus's motion is best described with average speed and with average velocity.

Average speed is calculated as the *total traveled distance* divided by the *total time traveled*.

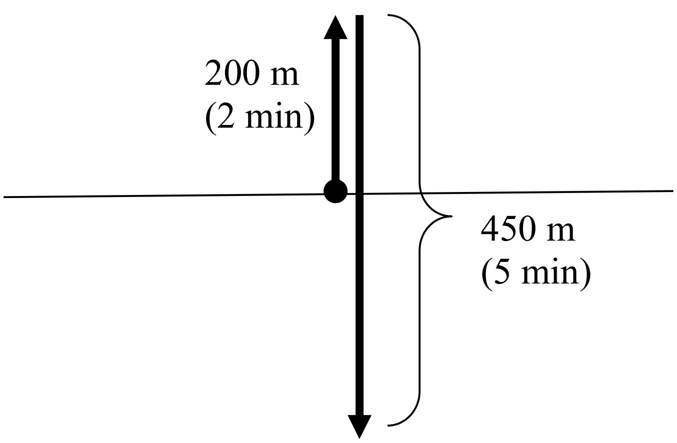
$$v_{avg} = \frac{\text{Total Distance}}{\text{Total Time}}$$

Average velocity is calculated as the *displacement* divided by the *total time traveled*. Remember, displacement is the absolute straight-line difference in position from where motion started to where motion ended.

$$\vec{v}_{avg} = \frac{\text{Displacement}}{\text{Total Time}}$$

The total time of travel includes all time between the start of motion and the end of motion. If the object stops or pauses within its travel, that time must be included in the total travel time.

Illustrative Example



A man walked 200 m north in 2 minute. He then walked 450 m south for another 5 minutes. Calculate the man's average speed and his average velocity. The solid arrows show how far the man walked (using coordinate lines for reference.) His total time of travel was 7 minutes.

The man walked a total distance of 650 m (200 m north, then another 450 m south). The man's displacement is 250 m S (the change in position between start and his final position).

To calculate the man's average speed

$$\bar{v} = \frac{\text{Total Distance}}{\text{Total Time}} = \frac{650\text{ m}}{420\text{ s}} = 1.55\,{}^{m}\!/_{s}$$

To calculate the man's average velocity

$$\bar{v} = \frac{\text{Displacement}}{\text{Total Time}} = \frac{250\text{ m}}{420\text{ s}} = 0.59\,{}^{m}\!/_{s}\ S$$

The man's average speed is 1.55 m/s and his average velocity is 0.59 m/s S. (Note that the total time of travel is 7 minutes.)

Illustrative Example

Monica walks from her house to the public library in 5 minutes. Monica walks 300 m east, turns the corner, then walks 300 m north. Calculate Monica's average speed and average velocity.

Monica's walked 300 m east, then another 300 m north for a total distance of 600 m. Her displacement (the straight-line change in position between her house and the library) was 424 m NE.

To calculate the average speed

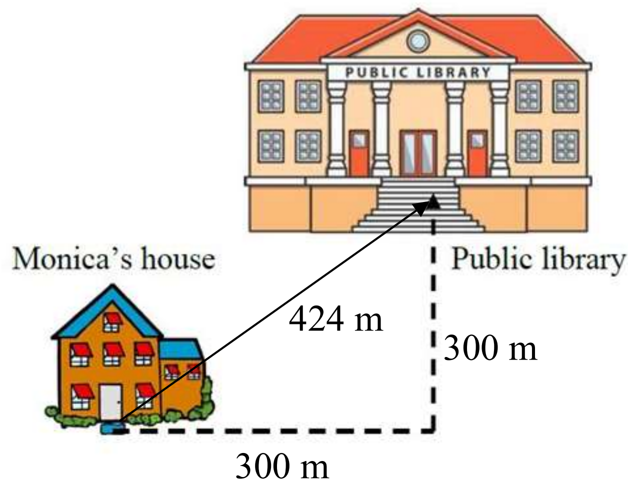
$$\bar{v} = \frac{\text{Total Distance}}{\text{Total Time}} = \frac{600\text{ m}}{300\text{ sec}} = 2.0\,{}^{m}\!/\!_{s}$$

Determine the displacement first

$$C = \sqrt{(300\,m)^2 + (300\,m)^2} = 424\,m\text{ NE}$$

Calculate average velocity

$$\bar{v} = \frac{\text{Displacement}}{\text{Total Time}} = \frac{424\text{ m}}{300\text{ s}} = 1.41\,{}^{m}\!/\!_{s}\ NE$$



Monica's average speed was 2.0 m/s and her average velocity was 1.41 m/s NE.

Resultant Velocity

A **resultant velocity** is the vector sum of all velocities acting upon an object. In other words, the resultant velocity is the "real" velocity of a moving object when that object has multiple motion components (vectors) or is being affected by other objects change its velocity. For example, if a car that moves in the same direction as the wind blows (tail wind), the car's resultant velocity is the sum of the car's velocity and the wind's velocity—the wind makes the car move faster. If a car that moves in the direction opposite as the wind blows (head wind), the car's resultant velocity is the difference between the car's velocity and the wind's velocity—the wind makes the car move slower.

How to calculate resultant velocity from vector components

- If the velocity vectors are in 1-dimension, add the velocity vectors together. Geographical and positional directions must have be positive or negative.
- If the velocity vectors are in 2-dimensions (right angles), use the Pythagorean Theorem.

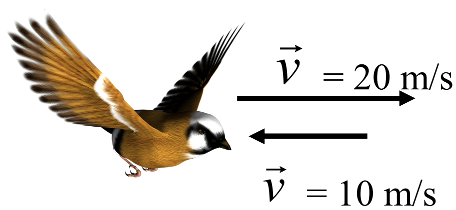
Example: A bird flies due east with a velocity of 20 m/s. A tail wind blows due east with a velocity of 10 m/s. Calculate the resultant velocity if the bird.

$$\vec{v}\,'_{Bird} = \vec{v}_{Bird} + \vec{v}_{wind}$$

$$\vec{v}\,'_{Bird} = 20\frac{m}{s} + 10\frac{m}{s} = 30\frac{m}{s}$$

The bird will fly with a resultant velocity of 30 m/s E. The wind pushes the bird to the east, the same direction that the bird already flies. As a result, the bird's velocity will be faster because of the wind's push.

Example: A bird flies due east with a velocity of 20 m/s. A headwind blows due west with a velocity of 10 m/s. Calculate the resultant velocity if the bird.



$$\vec{v}\,'_{Bird} = \vec{v}_{Bird} + \vec{v}_{wind}$$

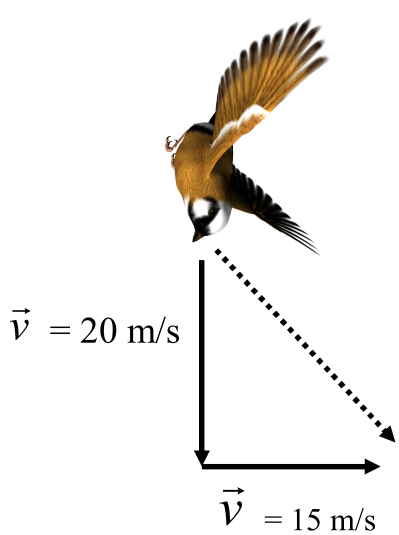
$$\vec{v}\,'_{Bird} = 20\frac{m}{s} + -10\frac{m}{s} = 10\frac{m}{s}$$

The bird will fly with a resultant velocity of 10 m/s east. The wind pushes the bird to the west, the opposite direction that the bird flies. As a result, the bird's velocity will be slower because of the wind's push in the opposite direction.

Example: A bird flies due south with a velocity of 20 m/s. A strong cross wind blows due east with a velocity of 15 m/s. Calculate the resultant velocity if the bird. (Note that the resultant velocity is the hypotenuse of the right triangle.

$$\vec{v}\,'_{Bird} = \sqrt{\left(\vec{v}_{Bird}\right)^2 + \left(\vec{v}_{wind}\right)^2}$$

$$\vec{v}\,'_{Bird} = \sqrt{\left(20\,{}^{m}\!/_{s}\right)^2 + \left(15\,{}^{m}\!/_{s}\right)^2} = 25\,{}^{m}\!/_{s}$$



The bird will fly with a resultant velocity of 25 m/s SE. The wind pushes the bird off course and increases the magnitude of its flying velocity.

PHYSICS: Lesson 05
ACCELERATION: CHANGING VELOCITY

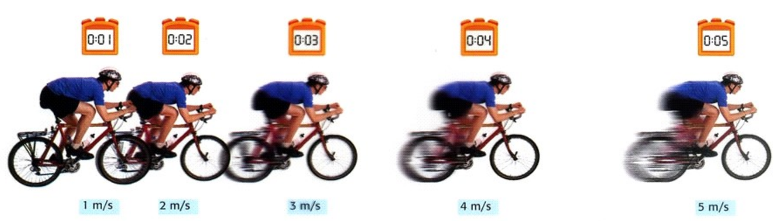
Learning targets

1. Students will calculate acceleration when given the variables distance and time, or change in velocities and time. (SP1.a)
2. Students can analyze figures, experimental data, graphs, and word problems and identify when objects are not accelerating, when objects get faster with time, and when objects get slower with time. (SP1.a; SP1.b)
3. Students can calculate the final velocity or the distance traveled by an accelerating object at the end of acceleration (SP1.a)
4. Students can calculate acceleration by using motion graphs of position vs. time and velocity vs. time. (SP1.b)

Acceleration is a change in velocity per unit time. *Forces cause acceleration*; *forces cause objects to change their states of motion*. Just like velocity, acceleration is a vector—it must have a magnitude (how big, the number) and a direction. Remember that ***velocity*** is defined as the straight line rate of motion in a given direction. Any time velocity changes, an acceleration has occurred.

Objects may experience one or more accelerations at any given time when acted upon by an unbalance force.

- An object's rate of motion may increase with time—get faster.
- An object's rate of motion may decrease with time—get slower.
- An object's direction may change with time—move in a curved path or in a circle.
- An object may experience both a change in rate (faster or slower) and a change in direction.



The cyclist is accelerating because his velocity is changing with time. Note that his velocity is progressively increasing by 1 m/s every second he rides.

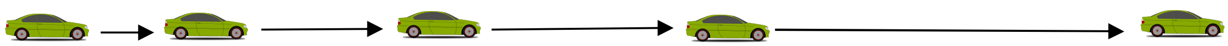
Describing Acceleration

If an object moves at constant velocity, the object moves equal distances in equal times. The spacing between the positions of the moving object at regular time intervals also remains constant. If an object moves at constant velocity, it is not accelerating; acceleration is zero.

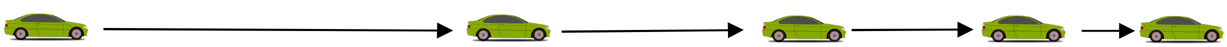
Constant velocity (no acceleration)

When objects accelerate, the velocity will increase (get faster) or decrease (get slower) with time. The spacing between the positions of the moving object at regular time intervals will get increasingly greater (as velocity increases) or get shorter (as velocity decreases).

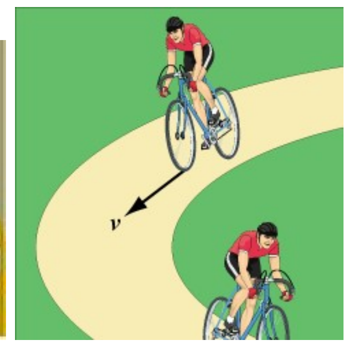
Increasing velocity (getting faster)



Decreasing velocity (getting slower)



Centripetal acceleration is the acceleration experienced by an object when it changes direction, moves in a curved path, or moves with circular motion. Centripetal acceleration describes how much change the object's velocity undergoes as the object changes direction.



Moving around a curve or in a circle is form of acceleration even if the object's magnitude of "how fast" remains constant. The acceleration occurs because the instantaneous velocity changes as the object turns.

All accelerations are in units of **m/s²** (meters per square seconds). Acceleration numbers and units should be interpreted as "the amount of increase or decrease in velocity per second of acceleration."

- Suppose a car is speeding up with an acceleration of 1.2 m/s^2. This means that for every second that the car is speeding up, its velocity increases by 1.2 m/s.
- Suppose a car is slowing down with an acceleration of -0.80 m/s^2. This means that for every second that the car is slowing, its velocity decreases by 0.80 m/s.

Acceleration is a ***vector***—it must have a magnitude and a direction. If a change in rate (how fast) is observed, identify (1) which direction is the object moving, and (2) identify if the object is getting faster with time or getting slower with time. If the object is getting faster with time, then a force is causing acceleration in the same direction that the object is moving—the force is speeding up the object. Thus, the acceleration is in the same direction as motion. If the object is getting slower with time, then a force is causing acceleration in the direction opposite that the object is moving. Thus, the acceleration is in the opposite direction as motion—the force is slowing down the object.

Examples of acceleration direction based on changes in velocity.

- Car moves north, velocity increases — The acceleration is north
- Car moves south, velocity decreases — The acceleration is north
- Rocket moves up, velocity increases — The acceleration is up
- Rocket moves up, velocity decreases — The acceleration is down

Calculating Acceleration

Solving for ***acceleration***. Use this equation if the problem states the change in velocity: the initial and the final velocities.

$$a = \frac{\vec{v}_f - \vec{v}_0}{t}$$

a = acceleration (m/s^2)
v_f = final velocity, (m/s)
v_o = initial velocity (m/s)
t = time (s)

Solving for ***acceleration***. Use this equation if the problem states the distance over which the object is speeding up or slowing down, and the initial velocity of the object.

$$a = \frac{2\cdot\left(\Delta x - \vec{v}_0 \cdot t\right)}{t^2}$$

a = acceleration (m/s^2)
v_o = initial velocity (m/s)
Δx = change in position, or distance (m)
t = time (s)

Example: From a stop (0 m/s), a car accelerates to 15 m/s in 18 seconds. Calculate the acceleration of the car.

0 m/s 18 s 15 m/s

The problem states the initial and final velocity of the car. Use the first acceleration equation.

$$a = \frac{v_f - v_0}{t} = \frac{15\,{}^{\text{m}}\!/_{\text{s}} - 0\,{}^{\text{m}}\!/_{\text{s}}}{18\,\text{s}} = \frac{15\,{}^{\text{m}}\!/_{\text{s}}}{18\,\text{s}} = 0.833\,{}^{\text{m}}\!/_{\text{s}^2}$$

Example: A car on the freeway changes velocity from 24 m/s to 18 m/s in 12 seconds. Calculate the acceleration of the car.

24 m/s 12 s 18 m/s

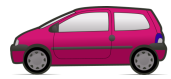
The problem gives the initial and final velocity of the car. Use the first acceleration equation.

$$a = \frac{v_f - v_0}{t} = \frac{18\,{}^{\text{m}}\!/_{\text{s}} - 24\,{}^{\text{m}}\!/_{\text{s}}}{12\,\text{s}} = \frac{-6\,{}^{\text{m}}\!/_{\text{s}}}{12\,\text{s}} = -0.50\,{}^{\text{m}}\!/_{\text{s}^2}$$

Example: Starting from stop at a traffic signal, a car accelerates uniformly for 60 seconds over a distance of 600 meters. Calculate the uniform acceleration of the car. Calculate the car's final velocity after acceleration is complete.

v_f ← - - - - - - - - - - - - - - - - - - 0 m/s

600 m; t = 1 min



The problem gives the initial velocity, the time of acceleration, and the distance over which the car accelerates. Use the second equation.

$$a = \frac{2 \cdot \left(\Delta x - \vec{v}_0 \cdot t\right)}{t^2} = \frac{2 \cdot \left(600\,\mathrm{m} - \left(0\,\mathrm{m}/\mathrm{s} \cdot 60s\right)\right)}{(60\,\mathrm{s})^2} = \frac{1200\,\mathrm{m} - 0\mathrm{m}}{3600\,\mathrm{s}^2} = 0.333\frac{m}{s^2}$$

Additional Kinematic equations

Solving for ***final velocity*** after acceleration is complete. Use this equation if the problem states the initial velocity, acceleration, and time over which acceleration occurred.

$$\vec{v}_f = \vec{v}_0 + a \cdot t$$

a = acceleration (m/s^2)
v_f = final velocity, (m/s)
v_o = initial velocity (m/s)
t = time (s)

Solving for ***acceleration*** after acceleration is complete. Use this equation if the problem states the distance over which the object is speeding up or slowing down, the initial velocity of the object, and the final velocity. No time.

$$a = \frac{\left(\vec{v}_f\right)^2 - \left(\vec{v}_0\right)^2}{2 \cdot \Delta x}$$

a = acceleration (m/s^2)
v_f = final velocity, (m/s)
v_o = initial velocity (m/s)
Δx = distance of acceleration (m/s)

Solving for change in position, or **distance**, over which an object accelerates.

$$\Delta x = \vec{v}_0 \cdot t + \frac{1}{2} a \cdot t^2$$

a = acceleration (m/s^2)
Δx = distance(m)
v_o = initial velocity (m/s)
t = time (s)