Name: ____ **Block:** _______

PHYSICS: UNIT 4
Momentum, Impulse, and Collisions

SP3. Obtain, evaluate, and communicate information about the importance of conservation laws for mechanical energy and linear momentum in predicting the behavior of physical systems.

a. Ask questions to compare and contrast open and closed systems.

d. Construct an argument supported by evidence of the use of the principle of conservation of momentum to
- explain how the brief application of a force creates an impulse.
- describe and perform calculations involving one dimensional momentum.
- connect the concepts of Newton's 3rd law and impulse.
- experimentally compare and contrast inelastic and elastic collisions.

Momentum	$p = m \cdot v$	p = momentum (kgm/s) m = mass (kg) v = velocity (m/s)
Impulse (Change in momentum caused by a force)	$J = \Delta p = p_f - p_0$ $J = \Delta p = m \cdot (v_f - v_0)$ $J = \Delta p = F \cdot t$ $J = \Delta p = m \cdot a \cdot t$	J = impulse (kgm/s) m = mass (kg) p_f = final momentum (kgm/s) p_0 = initial momentum (kgm/s) v_f = final velocity (m/s) v_0 = initial velocity (m/s) a = acceleration (m/s^2) F = reaction force (N) t = contact time (s)
Translational kinetic energy	$KE = \frac{1}{2} m \cdot v^2$	KE = kinetic energy (J) m = mass (kg) v = velocity (m/s)

PART 1: MOMENTUM & KINETIC ENERGY

Learning Targets

1. Students will calculate momentum and kinetic energy when provided with mass and velocity.
2. Students will compare and contrast momentums and kinetic energies of small, medium, and large mass objects moving at different velocities.
3. Students will use the KE equation to explain the exponential increase in kinetic energy with small increases in velocity.

MOMENTUM

Momentum is calculated as the product of mass and velocity. ***Mass*** is the quantity of matter in an object, and is proportional to the object's inertia. ***Velocity*** is how fast the object is moving in a straight line in a given direction. Velocity is a vector, therefore, momentum is also a vector. Momentum may be positive or negative depending on the direction of motion.

$$p = m \cdot v$$

p = momentum ($kg\frac{m}{s}$)

m = mass (kg)

v = velocity (m/s)

The units for momentum are the units of mass (kg) times the units of velocity (m/s): kilogram meter per second, $kg\frac{m}{s}$.

Momentum *is the "intensity of motion" of a moving object. Momentum describes the "moving inertia" of an object*. Remember, inertia is proportional to mass—the greater the mass of an object, the more inertia attributed to the object, the more the object will resist changing its state of motion when a force acts upon it. Momentum also considers how fast objects move (velocity). Larger objects that move very fast require a lot of force to change their states of motion because the momentum is so large. In contrast, smaller objects that move very slowly require very little force to change their states of motion because momentum is so small.

Momentum is the product of mass and velocity, therefore proportional to both velocity and to mass.

- The greater the mass of the moving object, the greater the momentum.
- The faster an object moves, the greater the momentum
- The greater the mass and velocity of the moving object, the greater the momentum.

Momentum is ***proportional*** to the mass of the moving object and proportional to the velocity of the moving object. The ability to change the velocity or state of motion of a moving object requires more and more force with (1) the greater the mass of the object and (2) the faster the object moves.

- The more massive the object that is in motion, *the greater the inertia attributed to the object,* the more difficult it will be to change the object's state of motion.
- The faster an object in motion moves, *the greater the kinetic energy attributed to the object*, the more work must be performed to slow or turn the object.

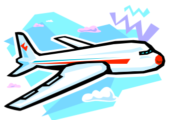
Illustrative Example

Calculate the momentum of a boy riding a skateboard. The combined mass of the boy and skateboard is 33 kg. The velocity of the boy and skateboard is 2.5 m/s.

$$p = m \cdot v$$

$$p = 33kg \cdot 2.5\, {}^{m}\!/\!{}_{s} = 82.5kg\,\frac{m}{s}$$

Illustrative example



Jet airplane: very large mass and very large velocity = very large momentum.

$$p = m \cdot v = 250{,}000kg \cdot 110\frac{m}{s} = 27{,}500{,}000\ kg\frac{m}{s}$$

Bumble bee: very small mass and low velocity = very low momentum.

$$p = m \cdot v = 0.008kg \cdot 4.0\frac{m}{s} = 0.032\ kg\frac{m}{s}$$

The effect of mass. Two different automobiles (of greatly different masses) are moving with the same velocity. In order to stop the truck, the brakes must exert at least 6-times the stopping force as the brakes on the car because the truck's mass and momentum is very large compared to the mass of the car. Because it takes longer for the truck to stop, the stopping distance of the truck is much greater than the car's stopping distance.

m = 500 kg, v = 20 m/s

$$p = 500kg \cdot 20\frac{m}{s} = 10{,}000\ kg\frac{m}{s}$$

m = 3000 kg, v = 20 m/s

$$p = 3000kg \cdot 20\frac{m}{s} = 60{,}000\ kg\frac{m}{s}$$

The effect of velocity. For example, two identical cars (same mass) are moving at two different velocities. In order to stop the car that is moving faster and has more energy, the brakes must perform more work as the brakes on the slower car because the faster car has twice the momentum of the first car.

m = 500 kg, v = 10 m/s

$$p = 500kg \cdot 10\frac{m}{s} = 5000\ kg\frac{m}{s}$$

m = 500 kg, v = 20 m/s

$$p = 500kg \cdot 20\frac{m}{s} = 10{,}000\ kg\frac{m}{s}$$

TRANSLATIONAL KINETIC ENERGY

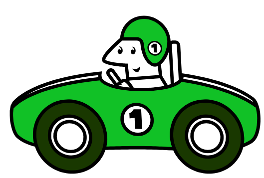
Translational kinetic energy (KE) is the energy attributed to an object in motion moving from one location to another. KE is calculated as ½ multiplied by the mass of the object (kg) multiplied by the velocity squared. KE is a scalar parameter and describes the moving energy of an object. Kinetic energy, like all forms of energy, is reported in units of Joules (J).

$$KE = \frac{1}{2}m \cdot v^2$$

KE = translational kinetic energy (joules)
m = mass of object moving (kg)
v = velocity of object in motion (m/s)

Illustrative Example

The racecar moves with a velocity of 42 m/s. The mass of the racecar is 700 kg. Calculate the translational KE of the racecar.



$$KE = \frac{1}{2} \cdot m \cdot v^2$$

$$KE = \frac{1}{2} \cdot 700kg \cdot \left(42\frac{m}{s}\right)^2$$

$$KE = 617{,}400\ J$$

Note that the velocity (v) in the kinetic energy equation is squared—as velocity increases, the quantity of KE exponentially increases. For example, if the speed of an object doubles, the KE attributed to that object quadruples. If the speed of an object triples, the KE attributed to that object increases nine-fold. A small increase in velocity will yield a very large increase in translational KE.

Illustrative example

Calculate and compare the skater's kinetic energies as his velocity increases.

Timmy has a mass of 40 kg. He rides his skateboard with a *velocity of 2.0 m/s*.

$$KE = \frac{1}{2}\cdot m\cdot v^2 = \frac{1}{2}\cdot 40kg\cdot \left(2.0\,{}^{m}\!/_{s}\right)^2$$
$$KE = 80J$$

Timmy skates faster down a hill with a *velocity of 4.0 m/s*.

$$KE = \frac{1}{2}\cdot m\cdot v^2 = \frac{1}{2}\cdot 40kg\cdot \left(4.0\,{}^{m}\!/_{s}\right)^2$$
$$KE = 320J$$

Timmy skates very fast down a very steep hill with a *velocity of 8.0 m/s*.

$$KE = \frac{1}{2}\cdot m\cdot v^2 = \frac{1}{2}\cdot 40kg\cdot \left(8.0\,{}^{m}\!/_{s}\right)^2$$
$$KE = 1280J$$

Timmy's original kinetic energy was 80 J when he moved at 2.0 m/s. When his velocity was doubled to 4.0 m/s, his kinetic energy quadrupled (increased 4-times) to 320 J. When his velocity was doubled again to 8.0 m/s, his kinetic energy was 4-times greater than his kinetic energy at 4.0 m/s and 16-times greater than his kinetic energy at 2.0 m/s.

PART 2: IMPULSE

In UNIT 1, ***acceleration*** was defined as the change in velocity caused by an unbalanced external force acting upon object. Forces cause acceleration. Any acceleration (change in velocity) will also cause a change in momentum because the velocity of the moving object will increase, decrease, or change direction. This sudden change in momentum due to forces causing an acceleration is called ***impulse***. Impulse is the result of a force acting upon an object (which causes acceleration) and must account for the contact time over which the force is applied. Mass, as always, is conserved and remains constant.

Unit 1, Acceleration is calculated as the change in velocity with time.

$$a = \frac{v_f - v_0}{t}$$

a = acceleration (m/s^2)
v_f= final velocity (m/s)
v_0 = initial velocity (m/s)
t = time (s)

Unit 2, Acceleration is also calculated as the force acting upon the object divided by the mass of the object.

$$a = \frac{F}{m}$$

m = mass (kg)
a = acceleration (m/s^2)
F = reaction force (N)

By substituting the equalities for acceleration and cross-multiplying the denominators, the equations for impulse Δp, or the change in momentum, can be derived.

$$\frac{F}{m} = \frac{v_f - v_0}{t} \Rightarrow \quad \Delta p = F\cdot t = m\left(v_f - v_0\right) = p_f - p_0$$

Impulse is calculated as the change in momentum. All four mathematical equations shown below calculate the identical value of impulse—they are equalities.

$$J = \Delta p = F \cdot t$$

$$J = \Delta p = p_f - p_0$$

$$J = \Delta p = m\left(v_f - v_0\right)$$

$$J = \Delta p = m \cdot a \cdot t$$

m = mass (kg)
p_f = final momentum (kgm/s)
p_0 = initial momentum (kgm/s)
v_f = final velocity (m/s)
v_0 = initial velocity (m/s)
a = acceleration (m/s^2)
F = reaction force (N)
t = contact time (s)

- Impulse = ***reaction force*** multiplied by the ***contact time***. The force caused the acceleration and the contact time was how long the force acted upon the object.
- Impulse = ***change in momentum*** of the object, the difference between the final momentum and the initial momentum.
- Impulse = ***the mass of the object multiplied by the change in velocity***.
- Impulse = mass multiplied by the acceleration multiplied by the acceleration time.

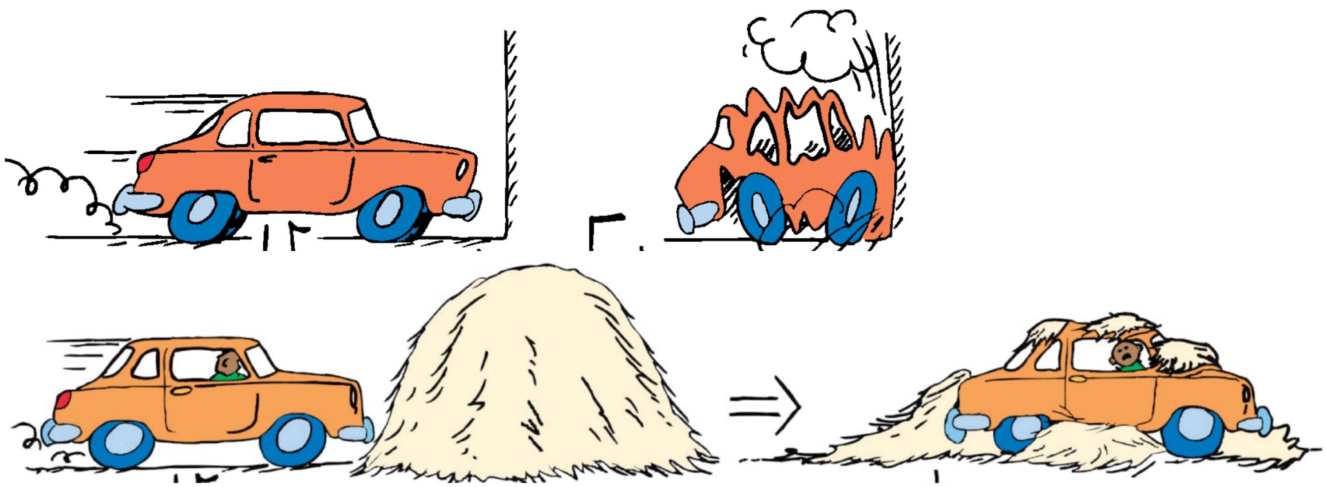
According the ***momentum-impulse theorem***, the application of an unbalanced force upon an object multiplied by the contact or reaction time over which the force is applied will result in the change in momentum (the impulse.) During a sudden change in momentum, *the time over which the acceleration occurs is inversely proportional to the magnitude of the force causing the acceleration*. If momentum (and velocity) changes instantaneously—for example, a moving object stops in a very short amount of time—the force causing the object to stop is very, very large. If momentum (and velocity) changes slowly over a longer time, the reaction force causing the impulse is much weaker.

Impulse is the product of the unbalanced force acting upon the object and the time at which the force is applied.

- The longer the reaction time over which the force acts (slower application of force), the weaker the force needed to cause the impulse.
- The shorter the reaction time over which the force acts (faster application of force), the stronger the force needed to cause the impulse.
- If the reaction time is instantaneous, the force is very, very strong.

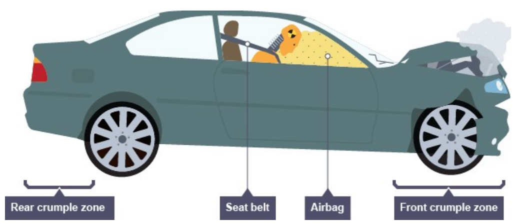
Illustrative Example

In both cases, the impulse is the same—both cars go from moving with a fast velocity to a stop. The outcomes of the impulse differ because the stopping forces are very different. The car in the top picture goes from moving very fast to an instantaneous stop by colliding with an immovable brick wall. The force of impact upon the car is very, very large because the car comes to a stop in a very short amount of time (in under 0.1 s). Very short time requires that the strength of the stopping force is very large. The damage to the car is extreme. In contrast, the car in the bottom picture goes from moving very fast to a stop by passing through a hay bale. The force of impact upon the car is very weak because the car comes to a stop in a much longer amount of time (~ 1.5 s). Longer time requires that the stopping force be very weak. The damage to the car is minimal.



Illustrative example: Modern Automobile Safety

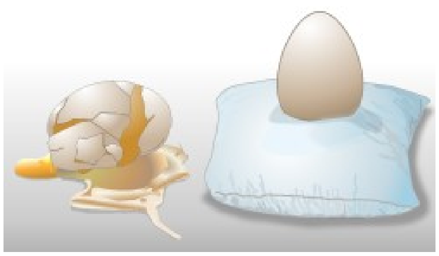
Modern automobiles are designed to reduce injury potential to passengers by employing seat belt restraints, airbags and crumple zones. During a collision, automobiles that are initially moving at high velocity (v_0 = very large) come to an unexpected violent stop (v_f = 0 m/s). These safety designs *extend the time of the impact* (to the car and to the passengers), thus reducing the force of the impact.



Front crumple zones in the front structure of the automobile purposefully crumple (fold inward and flatten) to absorb the force of the impact—the contact time between the initial collision and the automobile's stop is extended (t is longer). The front crumpling zones "absorb" the force of impact by allowing the cart to come to a stop over a longer time (1 second instead of 0.1 seconds). As a result, the force of the impact on the automobile's front end is reduced because the time to come to the stop is longer. The result is decreasing the damage to the automobile's structure.

Airbags and Seatbelts. Inside the cabin of automobile, the passenger's inertia wants to keep the body and head moving forward. The seatbelt's elasticity and "give" slows the passenger's body as the body snaps forward—again by extending the time over which the body jerks forward and reducing the force of the impact on the body. Airbags act as cushions for the head and upper body. When the airbag inflates, it forms a barrier between the driver or passenger's head and the car's dashboard or steering wheel. The cushioning of the airbag again extends the time of the body's collision with the interior of the car during the impact, reducing the force of impact on the head and body.

Illustrative example: The Egg Drop



If you drop a raw egg onto a hard surface, like a floor or pavement, the eggshell will break and the contents will splatter. The impulse occurs when the egg goes from freefall to the abrupt stop. Without any cushioning, the contact time with the hard surface and the egg during impact is very short (under 0.01 s) and the crush force acting on the eggshell is very large.

In contrast, if you drop a raw egg onto a pillow, the egg will not break. The pillow's spongy flexible filling will cushion the egg as it falls into the pillow, extending the contact time between when the egg first lands on the pillow to when it comes to rest. By extending the time for the egg to come to rest up to 0.5 s, the force of impact is much weaker, The impulses are the same, however, the force of impacts are very different.

Illustrative example

A boy rides his skateboard at with a velocity of 5.0 m/s. He jumps off of the skateboard to a stop in 0.5 seconds. His mass is 40 kg.

- Calculate the initial momentum.
- Calculate the final momentum.
- Calculate the acceleration.
- Calculate the impulse.
- Calculate the reaction force of the boy's legs causing the boy to stop.

Initial momentum

$$p_0 = m \cdot v_0 = 40kg \cdot 5.0\, {}^{m}\!/_{s} = 200\, kg\, {}^{m}\!/_{s}$$

Final momentum

$$p_f = m \cdot v_f = 40kg \cdot 0\, {}^{m}\!/_{s} = 0\, kg\, {}^{m}\!/_{s}$$

Acceleration

$$a = \frac{v_f - v_0}{t} = \frac{0\, {}^{m}\!/_{s} - 5.0\, {}^{m}\!/_{s}}{0.5s} = -10\, {}^{m}\!/_{s^2}$$

Impulse
(all of the calculations yield the same answer for impulse)

$$J = \Delta p = p_f - p_0 = 0\, kg\, {}^{m}\!/_{s} - 200\, kg\, {}^{m}\!/_{s} = -200\, kg\, {}^{m}\!/_{s}$$

or

$$J = \Delta p = m\left(v_f - v_0\right) = 40kg \cdot \left(0\, {}^{m}\!/_{s} - 5.0\, {}^{m}\!/_{s}\right) = -200kg\, {}^{m}\!/_{s}$$

or

$$J = \Delta p = m \cdot a \cdot t = 40kg \cdot -10\, {}^{m}\!/_{s^2} \cdot 0.5s = -200kg\, {}^{m}\!/_{s}$$

Force

$$F = \frac{J}{t} = \frac{-200\, kg\, {}^{m}\!/_{s}}{0.5s} = -400N$$

or

$$F = m \cdot a = 40kg \cdot -10\, {}^{m}\!/_{s^2} = -400N$$

Illustrative Example

The batter hits the baseball. Before impact, the baseball was moving at -37 m/s. After impact with the bat, the baseball was moving at +48 m/s. The contact time between the bat and the ball was 0.2 seconds. The baseball's mass was 0.15 kg.

- Calculate the initial and final momentum.
- Calculate the acceleration.
- Calculate the impulse.
- Calculate the reaction force of the baseball bat impacting the baseball

Initial momentum
Final momentum

$$p_0 = m \cdot v_0 = 0.15kg \cdot -37\, {}^{m}/_{s} = \text{-}5.55\, kg\, {}^{m}/_{s}$$

$$p_f = m \cdot v_f = 0.15kg \cdot 48\, {}^{m}/_{s} = 7.20\, kg\, {}^{m}/_{s}$$

Acceleration

$$a = \frac{v_f - v_0}{t} = \frac{48\, {}^{m}/_{s} - -37\, {}^{m}/_{s}}{0.2s} = 425\, {}^{m}/_{s^2}$$

Impulse
(all of the calculations yield the same answer for impulse)

$$J = \Delta p = p_f - p_0 = 7.20\, kg\, {}^{m}/_{s} - -5.55\, kg\, {}^{m}/_{s} = 12.75\, kg\, {}^{m}/_{s}$$

or

$$J = \Delta p = m\left(v_f - v_0\right) = 0.15kg \cdot \left(48\, {}^{m}/_{s} - -37\, {}^{m}/_{s}\right) = 12.75kg\, {}^{m}/_{s}$$

or

$$J = \Delta p = m \cdot a \cdot t = 0.15kg \cdot 425\, {}^{m}/_{s^2} \cdot 0.2s = 12.75kg\, {}^{m}/_{s}$$

Force

$$F = \frac{\Delta p}{t} = \frac{12.75\, kg\, {}^{m}/_{s}}{0.2s} = 63.7N$$

or

$$F = m \cdot a = 0.15kg \cdot 425\, {}^{m}/_{s^2} = 63.7N$$

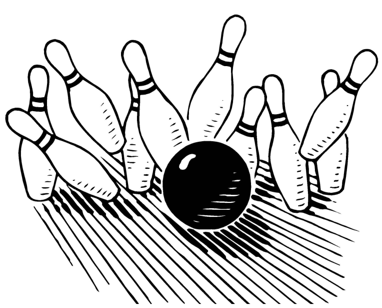
PART 3: ELASTIC AND INELASTIC COLLISIONS

Law of Conservation of Linear Momentum

Law of Conservation of Linear Momentum: *In a closed system, the total momentum among all objects is conserved; in a closed system, the sums of the momentums of all objects before they interact must equal the sums of momentums of all objects after they interact*.

In other words: When two or more objects collide, the sum of momentums of all objects before they collide must equal the sum of momentums of all objects after they collide. OR If objects interact, such as in a collision where two or more objects collide or bump into each other, the total momentum before the collision must equal the total momentum after collision.

The premise behind the law of conservation of momentum is to describe how momentum is transferred between objects when the objects collide without any momentum being lost—total momentum across all objects before the collision must equal the total momentum across all objects after the collision. Some objects will experience a decrease in momentum whereas some objects will experience an increase in momentum. When accounting for all of the momentums before the collision and after the collision, remember to use positive and negative signs to designate the direction of the velocity and the momentums.



Illustrative example

The momentum of the rolling bowling ball before the collision with the bowling pins EQUALS the sum of momentums of the bowling ball and the scattering bowling pins after the collision.

$$p_{before} = p_{after}$$

$$p_{BALL} = p_{BALL} + \Sigma p_{PINS}$$

The moving bowling ball transferred some of its momentum to the stationary pins, putting the bowling pins in motion.

Illustrative example

The momentum of the cue ball before it impacts the rack of pool balls EQUALS the sum of the momentums of the cue balls and the scattering pool balls after the collision.

$$p_{before} = p_{after}$$

$$p_{CUE} = p_{CUE} + \Sigma p_{BALLS}$$

The moving cue ball transferred some of its momentum to the stationary pool balls, putting them into motion.

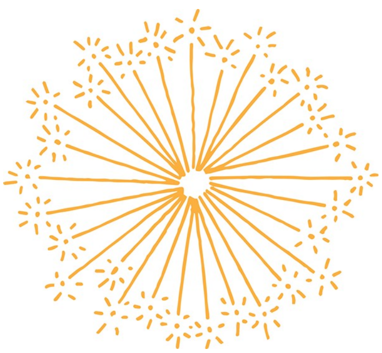
Illustrative example

At the carnival, the sum of the momentums of the two bumper cars before they collide EQUALS the sum of the momentums of the two bumper cars after they bounce apart.

$$p_{before} = p_{after}$$

$$p_{BC\#1} + p_{BC\#2} = p_{BC\#1} + \Sigma p_{BC\#2}$$

The interaction of the bumper cars caused momentum to be transferred from one bumper car to the other bumper car.



Illustrative example

A firework rocket explodes. Immediately before the explosion, the firework had a total momentum of zero. Immediately after the explosion, millions of fragments and embers scatter in every direction from the center of the explosion.

$$0kg\,{}^{m}\!/_{s} = p_{before} = p_{after}$$

$$0kg\,{}^{m}\!/_{s} = p_{Rocket} = \Sigma p_{Fragments}$$

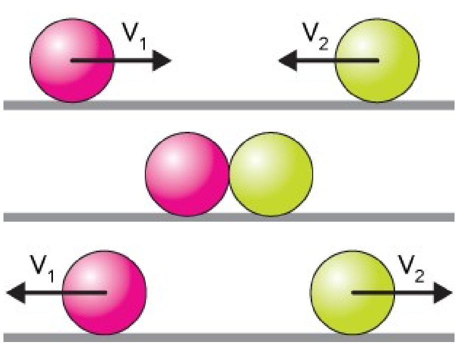
The vector sum of the momentums of the millions of fragments and embers will equal zero—they fly in a variety of positive and negative directions that cancel each other out when added together.

COLLISIONS

A ***collision*** is an interaction where two or more initially freely moving objects impact each other. During the collision, the objects may bounce off of each other, stick together, or physically deform each other.

Elastic collisions

Elastic collisions are perfect collisions. Elastic collisions occur when two or more rigid bodies collide and bounce off of each other perfectly. During an elastic collision, ***momentum is conserved*** **and** ***kinetic energy is conserved***. Kinetic energy is conserved because no energy is transformed into work (physical damage), heat or friction, or the "fusion" energy needed to fuse the bodies together.



(1) No physical damage between colliding bodies. No breakage, cracking, or change in shape.
(2) No energy loss due to friction (heat), light or sound caused by the impact.
(3) Bodies separate after the collision. The bodies do not merge or fuse together during the collision.

The "RULES" for elastic collisions

1. If two colliding bodies have **equal masses**, the bodies will **exchange momentums and exchange velocities** during the collision. This rule only applies if the masses of the two colliding bodies are equal. If the masses are unequal, velocities and momentums will not exchange.

2. During the collision, the body with the greater momentum before the collision happens will transfer some of its momentum to the object with the lesser momentum before the collision.

3. During the collision, the object with the lesser momentum before the collision will gain some momentum from the object with the greater momentum before the collision.

4. The object with the greater momentum before the collision will move slower after the collision than it did before the collision. The object with the lesser momentum before the collision will move faster after the collision than it did before the collision.

$$\text{Total } p \text{ Before} = \text{Total } p \text{ After}$$
$$\left(p_1 + p_2\right)_{Before} = \left(p_1 + p_2\right)_{After}$$
$$\left(m_1 \cdot v_1\right)_{before} + \left(m_2 \cdot v_2\right)_{before} = \left(m_1 \cdot v_1\right)_{after} + \left(m_2 \cdot v_2\right)_{after}$$

In a two body linear system, the sum of the momentums of the two bodies before the collision must equal the sum of the momentums of the two bodies after the collision. The variables on the left side of the equation represent the pre-collision velocity and momentums. The variables on the right side of the equation represent the post-collision velocity and momentums. The masses of the bodies never change.

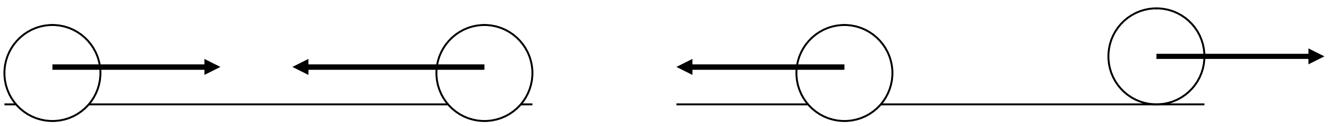
$$\text{Total } KE \text{ Before} = \text{Total } KE \text{ After}$$
$$\left(KE_1 + KE_2\right)_{Before} = \left(KE_1 + KE_2\right)_{After}$$
$$\left(\frac{1}{2}m_1 v_1^2\right)_{before} + \left(\frac{1}{2}m_2 v_2^2\right)_{before} = \left(\frac{1}{2}m_1 v_1^2\right)_{after} + \left(\frac{1}{2}m_2 v_2^2\right)_{after}$$

The sum of the kinetic energies of the two bodies before the collision must equal the sum of the kinetic energies of the two bodies after the collision. The variables on the left side of the equation represent the pre-collision kinetic energies. The variables on the right side of the equation represent the post-collision kinetic energies.

Illustrative Example

Two balls are approaching each other and collide. After the collision, the balls rebound in opposite directions. The sum of momentums of ball 1 and ball 2 before the collision must equal the sum of momentums after the collision.

Before the collision		After the collision	



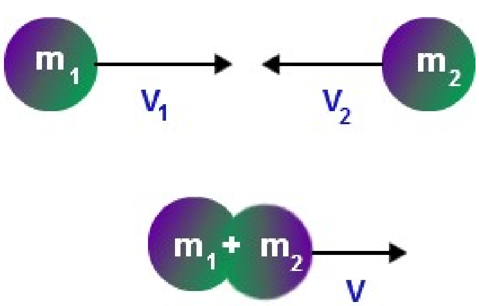
Ball 1	**Ball 2**	**Ball 1**	**Ball 2**
m = 2.0 kg	m = 3.0 kg	m = 2.0 kg	m = 3.0 kg
v = 2.0 m/s	v = -2.0 m/s	v = -2.8 m/s	v = 1.2 m/s
p = 4.0 kgm/s	p = -6.0 kgm/s	p = -5.6 kgm/s	p = 3.6 kgm/s
KE = 4.0 J	KE = 6.0 J	KE = 7.84 J	KE = 2.16 J

Ball 2 had the greater momentum and ball 1 had the lesser momentum before the collision. **Momentum was transferred from ball 2 to ball 1**. Ball 1 had a greater momentum and was moving faster after the collision than it did before the collision. Ball 2 had a lesser momentum and was moving slower after the collision than it did before the collision. The total momentum was conserved—the total momentum added together across all objects before and after the collision was -2.0 kgm/s respectively. The total kinetic energy was also conserved because no energy was lost as friction, heat, or damage during the collision. The total KE before the collision was 10 J and was also 10 J after the collision.

$$\text{Total p Before} = \text{Total p After}$$
$$4.0kg\, {}^{m}\!/_{s} + -6.0kg\, {}^{m}\!/_{s} = -5.6kg\, {}^{m}\!/_{s} + 3.6kg\, {}^{m}\!/_{s}$$
$$\text{Total } KE \text{ Before} = \text{Total } KE \text{ After}$$
$$4.0J + 6.0J = 7.84J + 2.16J$$

Inelastic collisions

Inelastic collisions are imperfect collisions. Inelastic collisions occur when two or more rigid objects collide and lose kinetic energy during the collision. Momentum is still conserved despite kinetic energy not being conserved.



(1) There may be physical damage between colliding objects like breakage, cracking, or changes in shape.
(2) Energy loss due to friction, or from heat at impact, or by the released of sound or light.
(3) The two objects may merge or fuse together to form 1 object.

The RULES for inelastic collisions (objects fuse together)

1. If two objects fuse or merge during the inelastic collision, the resultant mass is the sum of the masses of the two colliding objects. Add the masses together to get the fused mass.

2. The velocity of the two fused objects must be the mass-average of the two colliding objects before they collide—the velocity must be between the velocity of the faster object and the slower object before the collision.

3. The direction of motion of the two fused objects after the collision will be in the same direction as the object with the greatest momentum before the collision.

4. If two objects collide head-on, and have momentums of equal magnitude before the collision, the two fused objects will be motionless after the collision.

$$\text{Total } p \text{ Before} = \text{Total } p \text{ After}$$
$$(p_1 + p_2)_{Before} = (p_{12})_{After}$$
$$(m_1 \cdot v_1)_{before} + (m_2 \cdot v_2)_{before} = (m_{12} \cdot v_{12})$$

In a two body linear system, the sum of the momentums of the two bodies before the collision must equal the momentum of the two fused objects after the collision. The variables on the left side of the equation represent the pre-collision velocity and momentums. The variables on the right side of the equation represent the post-collision velocity and momentum if the two fused bodies. The masses of the bodies never change.

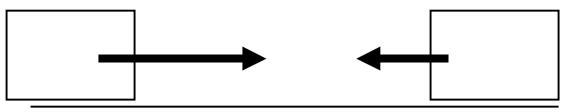
$$\text{Total } KE \text{ Before} > \text{Total } KE \text{ After}$$
$$(KE_1 + KE_2)_{Before} > (KE_{12})_{After}$$
$$\left(\frac{1}{2}m_1v_1^2\right)_{before} + \left(\frac{1}{2}m_2v_2^2\right)_{before} > \left(\frac{1}{2}m_{12}v_{12}^2\right)_{after}$$

In contrast, the sum of the kinetic energies of the two bodies before the collision must be greater than the kinetic energies of the fused bodies after the collision. Kinetic energy is lost because some of the original KE is converted into heat and "fusion" energy when the objects stick together. The variables on the left side of the equation represent the pre-collision kinetic energies. The variables on the right side of the equation represent the post-collision kinetic energy.

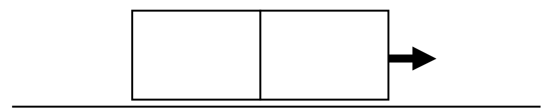
Illustrative Example

Two sliding boxes moving in opposite direction collide. After the collision, the two boxes are fuseded and move as one unit. The sum of momentums of box 1 and box 2 before the collision, 4.0 kgm/s, must equal the momentum of the fused boxes after the collision, 4.0 kgm/s. The sum of the kinetic energies of the box 1 and box 2 before the collision, 31 J, is greater than the kinetic energy of the fused boxes after the collision, 1.6 J.

Before the collision



After the collision



Box 1	**Box 2**	**Box 12**
m = 2.0 kg	m = 3.0 kg	m = 5.0 kg
v = 5.0 m/s	v = -2.0 m/s	v = 0.8. m/s
p = 10 kgm/s	p = -6.0 kgm/s	p = 4.0 kgm/s
KE = 25 J	KE = 6 J	KE = 1.6 J

$$\text{Total p Before} = \text{Total p After}$$
$$10.0kg\,{}^{m}\!/_{s} + -6.0kg\,{}^{m}\!/_{s} = -4.0kg\,{}^{m}\!/_{s}$$
$$\text{Total } KE \text{ Before} > \text{Total } KE \text{ After}$$
$$25J + 6.0J = 1.6J$$

Note that the final velocity of the fused boxes (+0.8 m/s) is the mass-average velocity of the original boxes' velocities (between +5.0 m/s and -2.0 m/s). Additionally, the direction of the fused object's motion is in the direction of the box 1's original motion because box 1 had the greater momentum before the collision.