Name: ____

PHYSICS: Unit 3
GRAVITATIONAL ATTRACTION & APPLIED FORCES

Georgia Standards of Excellence

SP1. Obtain, evaluate, and communicate information about the relationship between distance, displacement, speed, velocity, and acceleration as functions of time.

a. Plan and carry out an investigation of one-dimensional motion to calculate average and instantaneous speed and velocity.
- Analyze one-dimensional problems involving changes of direction, using algebraic signs to represent vector direction.
- Apply one-dimensional kinematic equations to situations with no acceleration, and positive, or negative constant acceleration.

d. Analyze and interpret data of two-dimensional motion with constant acceleration.
- Resolve position, velocity, or acceleration vectors into components (x and y, horizontal and vertical).
- Add vectors graphically and mathematically by adding components.
- Interpret problems to show that objects moving in two dimensions have independent motions along each coordinate axis.
- Design an experiment to investigate the projectile motion of an object by collecting and analyzing data using kinematic equations.
- Predict and describe how changes to initial conditions affect the resulting motion.
- Calculate range and time in the air for a horizontally launched projectile.

SP2. Obtain, evaluate, and communicate information about how forces affect the motion of objects.

b. Develop and use a model of a Free Body Diagram to represent the forces acting on an object (both equilibrium and non-equilibrium).

c. Use mathematical representations to calculate magnitudes and vector components for typical forces including gravitational force, normal force, friction forces, tension forces, and spring forces.

d. Plan and carry out an investigation to gather evidence to identify the force or force component responsible for causing an object to move along a circular path.
- Calculate the magnitude of a centripetal acceleration.

e. Develop and use a model to describe the mathematical relationship between mass, distance, and force as expressed by Newton's Universal Law of Gravitation.

Equations

Circumference of a circle	$C = 2 \cdot \pi \cdot r$	C = circumference (m) π = Pi = 3.14159 r = radius of the circle

Rotation speed or revolution speed	$v = \frac{n \cdot C}{t}$	v = rotation speed (m/s) C = circumference (m) n = number of times object rotates t = time (s)
Centripetal acceleration	$a_c = \frac{(v)^2}{r}$	a_c = centripetal acceleration (m/s^2) v = rotation speed (m/s) r = turn radius (m)
Centripetal force	$F_c = m \cdot a_c$	Fc = centripetal force (N) m = mass (kg) a_c = centripetal acceleration (m/s^2)
Freefall velocity	$v_f = v_0 - g \cdot t$	v_o = initial velocity (m/s) v = freefall velocity (m/s) g = acceleration in the gravity field (9.8 m/s^2) t = time of flight (s)
Freefall distance	$y = v_0 \cdot t - \frac{1}{2} g \cdot t^2$	h = freefall distance, height (m) g = acceleration in Earth's gravity field (9.8 m/s^2) t = time of flight (s)
Newton's Law of Universal Gravitational (Force of attraction)	$F_g = G \cdot \frac{m_1 \cdot m_2}{d^2}$	F_g = gravity force (N) G = Universal gravity constant $6.67 \times 10^{-11} N \frac{m^2}{kg^2}$ m = mass of objects (kg) d = distance between objects (m)
Weight	$w = m \cdot g$	w = weight (N) m = mass (kg) g = acceleration in a planet or moon's gravity field (m/s^2)
Gravity field on a planet or moon	$g = G \frac{M}{r^2}$	g = acceleration in a planet or moon's gravity field (m/s^2) G = Universal gravity constant $6.67 \times 10^{-11} N \frac{m^2}{kg^2}$ M = mass of the planet or moon (kg) r = radius of the planet or moon (m)

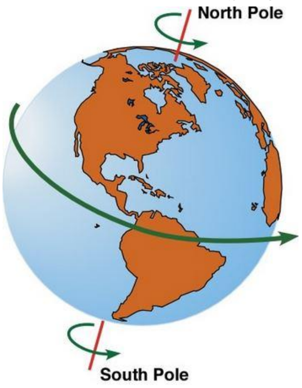
PART 1: UNIFORM CIRCULAR MOTION

Learning Targets

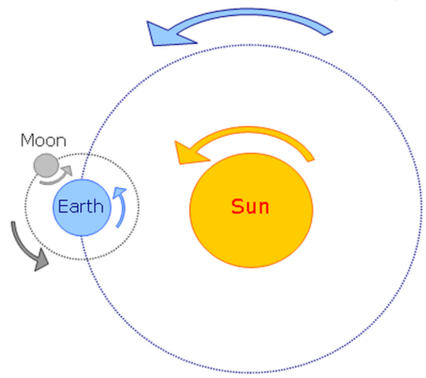
1. Students will calculate rotation speed, centripetal acceleration, and centripetal force acting upon a rotating or revolving body.
2. Students will analyze how manipulating the rotation speed and turn radius will increase and/or decrease the magnitude of centripetal acceleration and force.
3. Students will use vector diagrams (with acceleration and tangential velocity) to describe how circular motion is the resultant of those two perpendicular vectors.

Uniform circular motion is when an object moves at a constant rate in a circular path. Rotation and revolution are two different types of uniform circular motion. ***Rotation*** is the motion of an object spinning around its center of mass. ***Revolution*** is the motion of one object moving around another object. The Earth demonstrates both rotation and revolution motions. The Earth rotates on its axis, one rotation every 24 hours (day-night cycle). The Earth revolves around the sun, one revolution every 365.24 Earth days (year cycle). Likewise, Earth's moon revolves around the Earth, one revolution every 27 days.

Earth's rotation



Earth's revolution



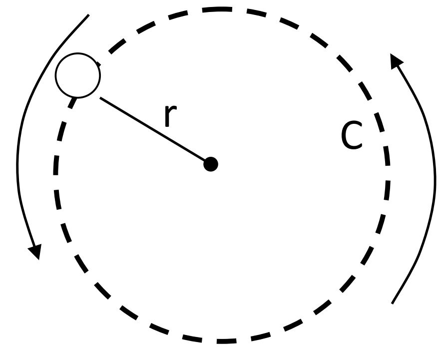
CALCULATING PROPERTIES OF LINEAR CIRCULAR MOTION

Objects that move with uniform circular motion have the following properties

- **turn radius** and **circumference**: physical dimensions of the circle
- **rotation speed** or **revolution speed**: how fast an object moves in a circular path
- **tangential velocity**: straight-line inertia effect of rotation speed; instantaneous velocity in a straight line parallel to the curved circumference of the circle.
- **centripetal acceleration**: "Center seeking acceleration", the change in velocity due to change in direction
- **centripetal force**: "Center seeking force", the force moving the object in a circular path.

Circumference

All circular paths are characterized by the radius (r) and the circumference. ***Radius*** **r** is ½ the ***diameter*** of the circle. The ***circumference*** of the circle (C) is the total linear distance around the circle's perimeter. Circumference is calculated from the radius using the equation



$$C = 2 \cdot \pi \cdot r$$

Circumference of the circle equals 2 times the product of pi and radius. The units for radius and circumference must be the same, preferably in meters. According to circle geometry, there are 2π radians (a measure of angles) for every 1 circumference completed (or 360º of angle).

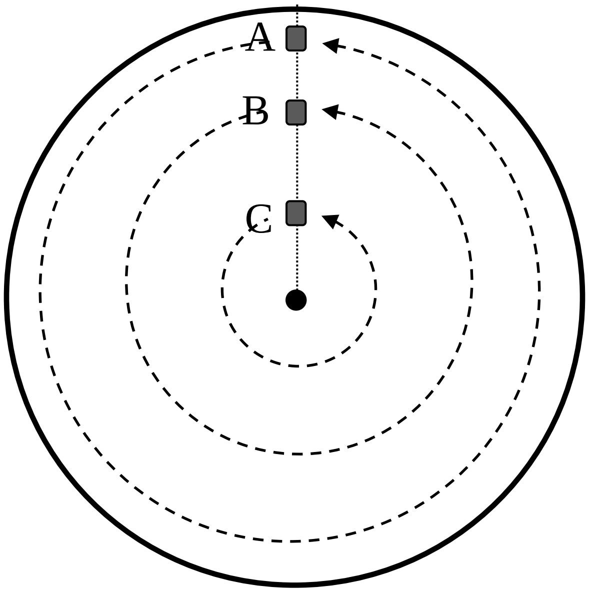
Linear rotation speed

Linear rotation speed is the linear circular speed at which an object rotates or spins around its center of mass, and is based on "how far" an object travels in a circular path per time. ***Revolution speed*** is the linear circular speed at which an object revolves or moves around another object. Both are speeds. Remember that speed is calculated as distance traveled per unit time—the rate at which an object moves.

$$v = \frac{n \cdot C}{t}$$

v = rotation or revolution speed (m/s)
n = number of times the object rotates or revolves.
C = circumference of the circular path (m)
t = time to complete the rotations or revolutions (s)

To calculate the linear rotation or revolution speed of an object in uniform circular motion, the circumference (C) of the circular path (circle length), the number of times the object rotates/revolves (n), and the time of travel must be known (t). The equation below is used to calculate the rotation or revolution speed. It is best to report rotation or revolution speed in units of m/s unless specified.

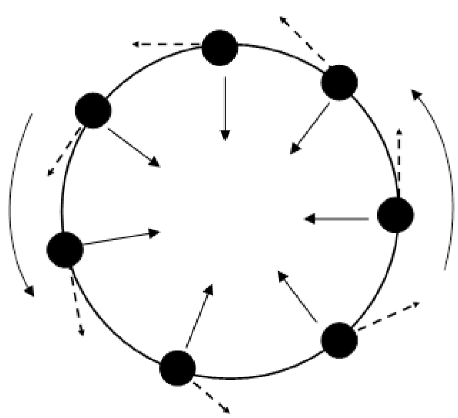


The diagram to the left shows a giant rigid rotating disk with three people standing on the disk (A, B, and C). If the disk rotated 1 time in 10 seconds, all three people riding on the disk experienced equal rotation in terms of moving in a circle.In other words, they all turned 360º—a full circle—in 10 seconds.

Conversely, the three people experienced very different linear rotation speeds. Person C had the slowest linear rotation speed because he was closest to the center of rotation (shortest local radius and smallest circumference to move). Person A had the fastest linear rotation speed because he was farthest from the center of rotation (longest local radius and longest circumference to move).

Centripetal acceleration

Acceleration is defined as a change in velocity with time. When forces accelerate objects, the observed accelerations may be changes in how fast the object moves (getting faster or getting slower with time) or changes in direction (turning) with time. ***Centripetal acceleration*** is the measurable change in velocity due to the constant change in direction experienced by an object in motion. The diagram shows an object moving in uniform circular motion. At each instant in time, the object changes direction, thus changing velocity.



The dashed vector arrows show the ***instantaneous velocity***—or the ***tangential velocity*** due to inertia—that the object is moving at various positions in its motion. Objects want to keep moving in a straight line. However, another force is simultaneously pulling inward to keep the object moving in a circular path.

The solid, inward pointing vector arrows show the direction of the centripetal acceleration due to object's curving motion. The magnitude of the centripetal acceleration remains constant but the direction constantly changes because centripetal acceleration always points inward toward the center of the circle regardless of its position.

$$a_c = \frac{\left(v^2\right)}{r}$$

a_c = centripetal acceleration (m/s^2)
v = rotation or revolution speed (m/s)
r = turn radius (m)

The equation to calculate centripetal acceleration contains two variables: the rotation/revolution speed (v) and the turn radius of the circle (r). Note that the rotation/revolution speed in the numerator is SQUARED. Note that the turn radius is in the denominator.

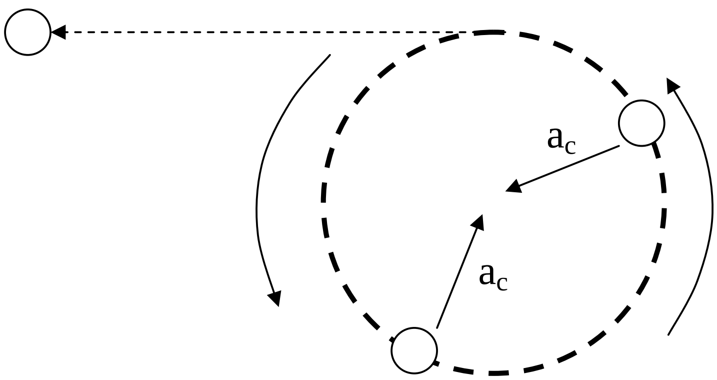
Centripetal acceleration is very sensitive to rotation/revolution speed because rotation speed in the numerator is squared. For example, a car moving around a curve at 10 m/s experiences 4-times the centripetal acceleration as a car moving at 5 m/s around the same curve. A car moving around a curve at 15 m/s experiences 9-times the centripetal acceleration as a car moving at 5 m/s around the same curve. Centripetal acceleration is also sensitive to turn radius. Generally speaking, the greater the turn radius (bigger the circle or bigger the curve), the lesser the centripetal acceleration will be because the denominator in the centripetal acceleration equation gets larger.

Centripetal force

Centripetal force is the inward "force" required to maintain an object moving in circular motion or in a curved path. Centripetal force causes centripetal acceleration. Just like all forces, centripetal force is equal to the product of mass and acceleration, where the acceleration is the centripetal acceleration and the mass is the mass of the object moving in a circle.

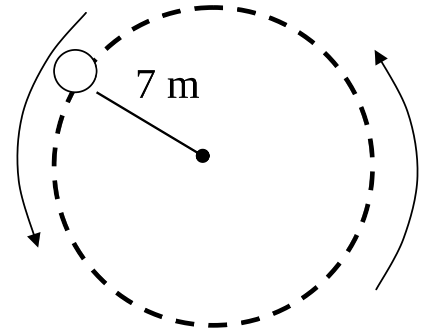
$$F_c = m \cdot a_c$$

F_c = centripetal force (N)
a_c = centripetal acceleration (m/s^2)
m = mass of object in circular motion (kg)



The direction of centripetal acceleration and centripetal force acting upon the object points inward to the center of the circle. If the inertia of the object in motion is greater than the centripetal force keeping, the object will move in a straight line direction at a constant velocity equal to the speed at which it moved in the circular path. This straight-line velocity is called the ***tangential velocity***. The tangential velocity is equal to the instantaneous velocity of the rotating or revolving object at the instant it is released. This can be demonstrated by swinging a ball on a string. As long as you hold onto the string, the ball at the end of the string will move in a circular path. If you let go of the string, the ball and string will fly away in a straight line with tangential velocity under the object's inertia.

Illustrative example



A merry-go-round has a radius of 7 meters. The merry go-round rotates 5 times per minute. A 20 kg boy is riding the hobby horse at the edge of the merry-go-round.

a. Calculate circumference. The radius of the merry-go-round is provided for you, 7 meters. Circumference of the merry-go-round is 44.0 meters.

$$C = 2 \cdot \pi \cdot r$$

$$C = 2 \cdot \pi \cdot 7m = 44.0m$$

b. Calculate rotation speed. The number of rotations is 5. The time to complete 5 rotations is 1 minute = 60 seconds. The circumference was calculated in the previous step. The rotation speed is 3.67 m/s.

$$v = \frac{n \cdot C}{t}$$

$$v = \frac{5 \cdot 44m}{60s} = 3.67 \, {}^{m}\!/_{s}$$

c. Calculate centripetal acceleration. The rotation speed was calculated in the previous step. The turn radius of the circle was 7 meters. The centripetal acceleration is 1.92 m/s^2.

$$a_c = \frac{(v)^2}{r}$$

$$a = \frac{\left(3.67 \, {}^{m}\!/_{s}\right)^2}{7m} = 1.92 \, {}^{m}\!/_{s^2}$$

d. Calculate centripetal force. The centripetal acceleration was calculated in the previous step. The mass of the boy is 20 kg. The centripetal force keeping the boy moving in a circle on the merry-go-round is 38.4 N.

$$F_c = m \cdot a_c$$

$$F = 20kg \cdot 1.92 \, {}^{m}\!/_{s^2} = 38.4N$$

PART 2: GRAVITATIONAL ATTRACTION FORCE

Learning Targets

1. Students can explain in their own words how changes in distance between two objects and mass of the objects affects the magnitude of gravitational attraction force.
2. Students can explain in their own words why gravity force is universal.
3. Students can predict which combination of objects, when given masses and distance between them, has the strongest and weakest gravitational attraction force.
4. Students can calculate the magnitude of gravitational attraction force using Newton's Law of Universal Gravitation equation.

Gravity is one of the four ***fundamental forces***. ***Gravity*** is a mutual attractive force (pull force) where objects naturally attract all other objects with mass. Gravitational attraction force is also an ***action-at-a-distance force***—objects do not need to be touching each other to pull on each other through gravity. ***Gravity is universal***: all objects pull on every other object in our universe by gravity force regardless of the distance separating them. Likewise, the other objects in our universe pull back. The ***gravity fields*** surrounding small objects—such as a person or an atom—is very weak. In contrast, the gravity fields surrounding larger, more massive objects—such as stars, planets, and moons—are very strong. Because planets like Earth are very massive compared to the objects on their surfaces, and because planets have immense inertia, unsuspended objects always freefall in the down direction toward the planet's surface. Planets never fall up, they have too much inertia to move.

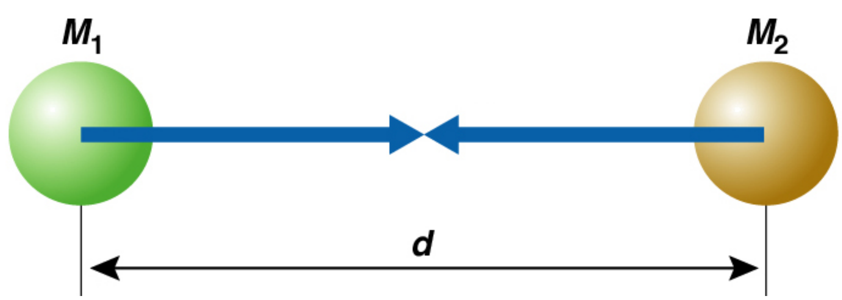
Newton's Law of Universal Gravitation: The gravitational attractive force between two objects is proportional to the products of their masses and inversely proportional to the square of the distance between the object's centers of mass.

The mathematical representation of Newton's Law of Universal Gravitation is shown below. The universal gravitational constant G is simply a number that relates the strength of gravitational force to mass and distance. G is NOT a mathematical equation, but an empirical constant that relates the masses of the attracting object to the distance between them.

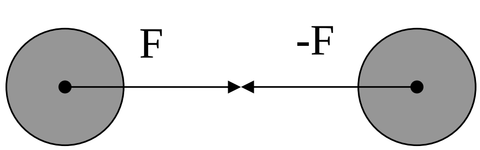
$$F_g = G \cdot \frac{m_1 \cdot m_2}{d^2}$$

$$G = 6.67 \times 10^{-11}\, N \frac{m^2}{kg^2}$$

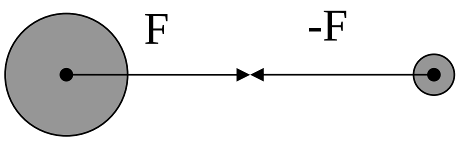
F_g = attractive force due to gravity (N)
m_1 = mass of object 1 (kg)
m_2 = mass of object 2 (kg)
d = distance between centers of mass of object 1 and object 2 (m)
G = universal gravitational constant



Gravity is a ***mutual attractive force*** (pull force) because it obeys ***Newton's 3rd Law of Motion***. Any two objects, regardless of size or mass difference or distance apart, will gravitationally attract each other with equal and opposite forces.



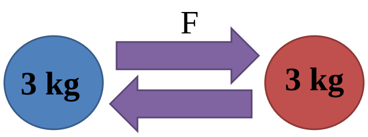
For example: two objects of equal mass will attract each other with an equal and opposite force. The object on the right will attract the object on the left with a gravity attraction force equal in magnitude and opposite in direction to the gravity attraction force of the object on the left.



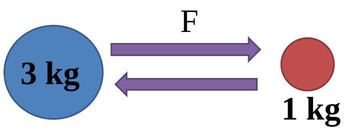
Two objects of unequal mass will attract each other with equal and opposite force despite the mass difference or size difference. The smaller object on the right attracts the larger object on the left with a force equal in magnitude and opposite in direction to the force the object on the left. Size does not matter, the gravity attraction is the same magnitude, but opposite in directions.

The Effect of Mass of Attracting Objects

The numerator of the fraction is the product of the masses of object 1 and object 2—the masses of the objects attracting each other. The ***product of the masses of the objects is proportional to the force***. All masses must be in kg.



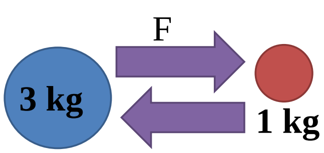
- The greater the product of the masses of the objects, the stronger the gravitational attractive force between them.



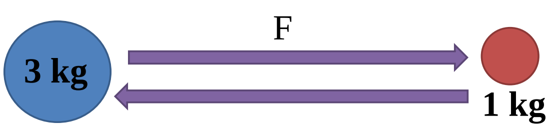
- The lesser the product of the masses of the objects, the weaker the gravitational attractive force between them.

The Effect of Distance Separating Attracting Objects

The denominator of the fraction is the distance between object 1 and object 2, and it is squared. This is an example of an inverse square law: the ***force is inversely proportional to the distance squared***. All distances must be in meters.



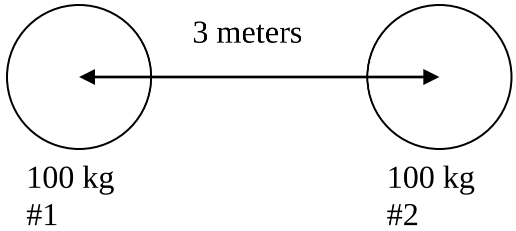
- The shorter the distance between the objects, the stronger the force between them.



- The greater the distance between the objects, the weaker the force between them.

Be aware that distance is squared in the denominator—it is a power relationship. Double the distance between objects, gravity force decreases to ¼ the original force. Triple the distance between objects, gravity force decreases to 1/9 the original.

Illustrative Example



The distance between the centers of gravity of two barrels of water is 3 meters. The mass of barrel 1 is 100 kg. The mass of barrel 2 is 100 kg. Calculate the gravitational attraction force between barrel 1 and barrel 2.

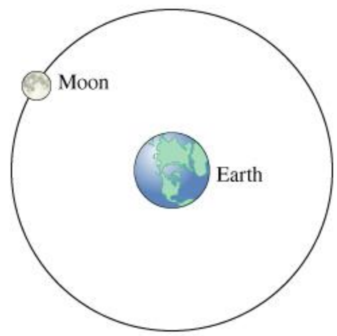
$$F_g = G \cdot \frac{m_1 \cdot m_2}{d^2} = \left(6.67 \times 10^{-11} N \frac{m^2}{kg^2}\right) \cdot \frac{100kg \cdot 100kg}{(3m)^2}$$

$$F_g = 7.41 \times 10^{-11} N$$

Illustrative example

The average distance between the Earth and Earth's moon is ~385,000 km = 385,000,000 m. The mass of the Earth is 5.98×10^{24} kg. The mass of Earth's moon is 7.35×10^{22} kg. Calculate the gravitational attraction force between the Earth and Earth's moon.

The gravitational attraction force between the bodies is very strong, 1.98×10^{20} N, because the objects are very massive and relatively close together (for celestial objects)



$$F_g = G \cdot \frac{m_1 \cdot m_2}{d^2} = \left(6.67 \times 10^{-11} N \frac{m^2}{kg^2}\right) \cdot \frac{5.98 \times 10^{24} kg \cdot 7.35 \times 10^{22} kg}{(3.85 \times 10^{8} m)^2}$$

$$F_g = 1.98 \times 10^{20} N$$

PART 3: PLANETARY GRAVITY AND WEIGHT

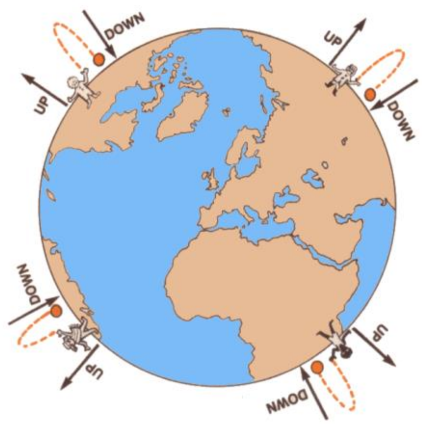
Learning Targets

1. Students should explain in their own words how and why gravity field strengths, escape velocities, and weights will differ on different bodies in our solar system.
2. Students should be able to calculate the weights of objects on Earth and on the surface of different celestial bodies when provided with values of g.
3. Students understand and explain why astronauts, people on amusement park rides, and people riding on airplanes often experience the sensation of weightlessness without being truly weightless.

Gravity fields

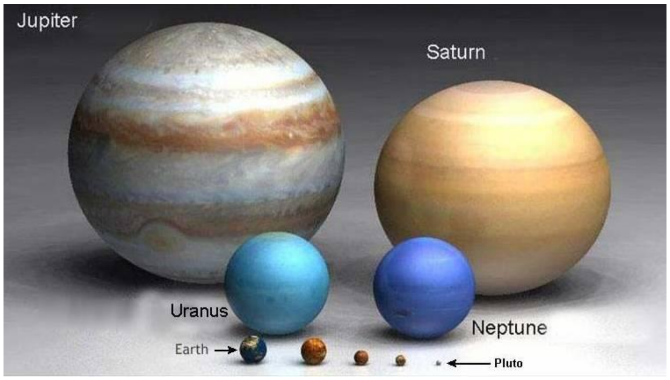
A ***gravity field*** is the physical space around an object in which gravitational attraction force between that object and other objects is strong enough to cause the objects to accelerate or move together. Gravity is the weakest fundamental force, however, gravity can become very strong if the masses of objects are very large or if objects are very close together. The gravity field surrounding a small object—such as a human or a pebble or an atom—is very weak. The gravity field surrounding a larger, more massive object—such as a planet, star, or moon—is very strong.

Planets and moons are very massive will have strong gravity fields due to their size and mass. Additionally, planets and moons have immense inertia (because of their very large masses). Despite gravity being a mutual pull force, the strong gravity of planets and moons coupled with the planet's or moon's inertia always results in the downward pull on the much smaller objects—e.g. resulting in freefall. Small objects always fall down toward the planet's surface because the planet's inertia is trillions of times greater and the smaller objects have such little mass and miniscule inertia. The planet never falls up toward the smaller object, the smaller object always falls down to the planet's surface. Additionally, the gravity field accelerates objects downward toward the Earth's surface on all sides of the planet as shown in the pictures.

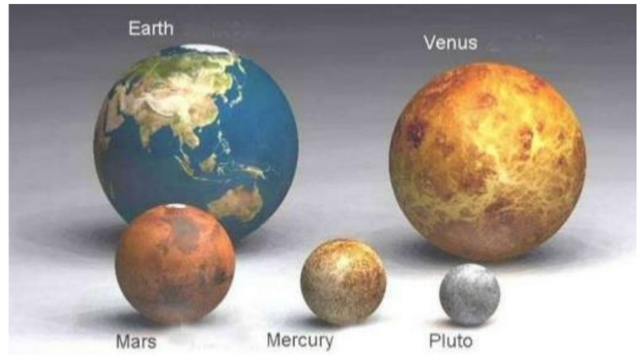


The two factors that affect the acceleration in a planet or moon's gravity field are (1) the mass of the planet or moon, and (2) the geometric radius of the planet or moon. The strength or magnitude of the gravity field increases proportionately with mass—bigger mass, stronger gravity field. Jupiter's gravity field is ~2.5 times greater than Earth's gravity field because Jupiter has a greater mass than the Earth. Likewise, the Earth's gravity field is ~6-times stronger than the moon's gravity field because Earth's mass is much greater than the moon's mass.

Body	**g (m/s^2) at surface**
Earth	9.81
Moon	1.62
Mercury	3.70
Mars	3.71
Jupiter	24.8
Sun	274

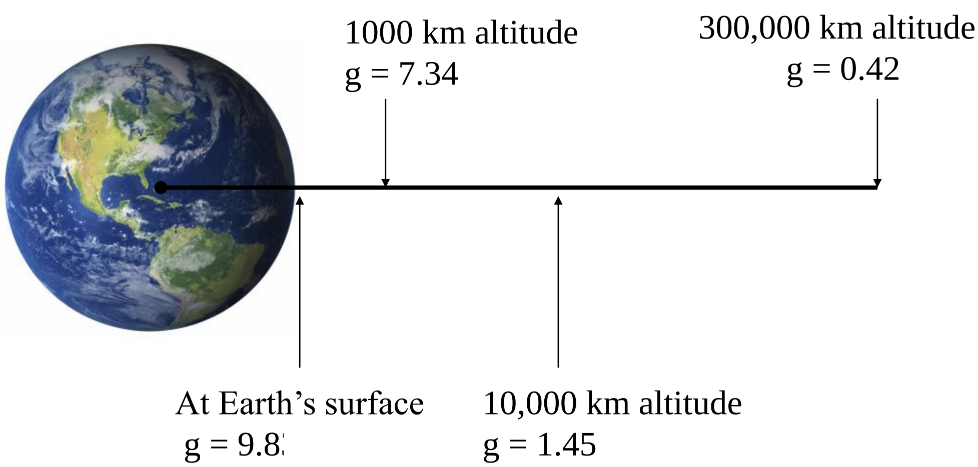


All planets in our solar system, compared for size. The large planets (gas and ice giants) have the strongest gravity fields because they have the greatest masses.



The terrestrial planets and Pluto, compared for size. The smaller planets have comparatively weaker gravity fields because they have much smaller masses.

Conversely, as the geometric radius away from the planet's center of mass increases, the gravity field becomes weaker—the farther away an object is from the geometric center of the planet or moon, the weaker the gravity field acting upon that object becomes. The picture on the next page shows the strength of the Earth's gravity field at different distances away from Earth's surface. The gravity field is the strongest at the Earth's surface with a magnitude of 9.8 m/s^2. The weights of objects are heaviest at the surface. Conversely, Earth's gravity field becomes weaker and weaker with increasing altitude. An airplane on the Earth's surface experiences a slightly stronger gravitational attraction to the Earth than the same airplane flying at 8000 m altitude above the surface. A satellite orbiting the Earth at 1000 km (1,000,000 m) above the Earth's surface would experience significantly weaker gravity because its radial distance between the Earth's geometric center. Regardless, there is still gravity and gravitational attraction—in space gravity may be weak but it is still present!



The gravity field around Earth gets weaker and weaker with increasing distance away from the Earth's surface.

Calculating the Magnitude of a Gravity Field

$$g = G \cdot \frac{M}{r^2}$$

$$G = 6.67 \times 10^{-11} N \frac{m^2}{kg^2}$$

g = acceleration in the gravity field (m/s^2)
M = mass of object; mass of planet (kg)
r = radius of object (m)
G = universal gravitational constant

Illustrative examples

Example 1: The planet Wheeler 424 has a mass of 6.00x10^{25} kg and a radius of 18,000 km. Calculate Wheeler 424's surface gravity.

$$g = G \cdot \frac{M}{r^2}$$

$$g = 6.67 \times 10^{-11} N \frac{m^2}{kg^2} \cdot \frac{6.00 \times 10^{25} kg}{(18{,}000{,}000 m)^2} = 12.3 \, {}^{m}\!/\!{}_{s^2}$$

Wheeler 424's surface gravity is 12.3 m/s^2. It is stronger than Earth's gravity.

Example 2: A satellite circles the planet Mars at a distance of 1000 km from Mars's surface. The satellite then lands on Mars's surface. Mars's mass is 6.39x10^{23} kg. Mars's radius is 3396 km.

- Calculate the acceleration of Mars's gravity field at the surface.
- Calculate the acceleration of Mars's gravity field at 1000 km above Mars's surface. (r = 4959 km)

At Mars's surface

$$g = 6.67 \times 10^{-11} N \frac{m^2}{kg^2} \cdot \frac{6.39 \times 10^{23} kg}{(3{,}396{,}000 m)^2} = 3.70 \, {}^{m}\!/\!{}_{s^2}$$

1000 km above Mars's surface

$$g = 6.67 \times 10^{-11} N \frac{m^2}{kg^2} \cdot \frac{6.39 \times 10^{23} kg}{(4{,}396{,}000 m)^2} = 2.20 \, {}^{m}\!/\!{}_{s^2}$$

On Mars's surface, the acceleration is 3.70 m/s^2. At 1000 m above the surface, the acceleration decreased to 2.20 m/s^2.

Weight

Mass is the quantity of matter contained in an object. ***Mass is always conserved***. Mass is independent of all forces and motions affecting it, and mass never changes based on location or motion. Conversely, weight will change based on its location because weight is dependent on the planet's gravity field strength. ***Weight*** is the measure of heaviness of an object in a planet's or moon's gravity field. Weight is the ***downward force*** upon an object that results from a planet's or moon's gravity field accelerating the object in the downward direction. Weight is calculated as the product of the object's mass and the downward acceleration in the planet's gravity field.

Weight is a force, and like all forces, weight is reported in units of Newtons. Weight is calculated as the product of the mass multiplied by the gravity field acceleration (g) for the planet or moon. Weight is reported as a positive number despite the acceleration due to gravity in the down direction.

$$w = m \cdot g$$

w = weight (N)
g = acceleration due to the gravity field (m/s^2)
m = mass of the object (kg)

For example, suppose a spaceship launches from Earth to the moon. The weight of spaceship on Earth will heavier than its weight on the moon because the Earth's gravity field is much stronger than the moon's gravity field. Earth is larger, more massive body, thus has a stronger gravity field compared to the moon. The spaceship will, however, have the same mass on both bodies because it is the same spaceship made of the same amount of matter.

Example 1: Calculate the weight of the car on Earth. The car's mass is 1200 kg.

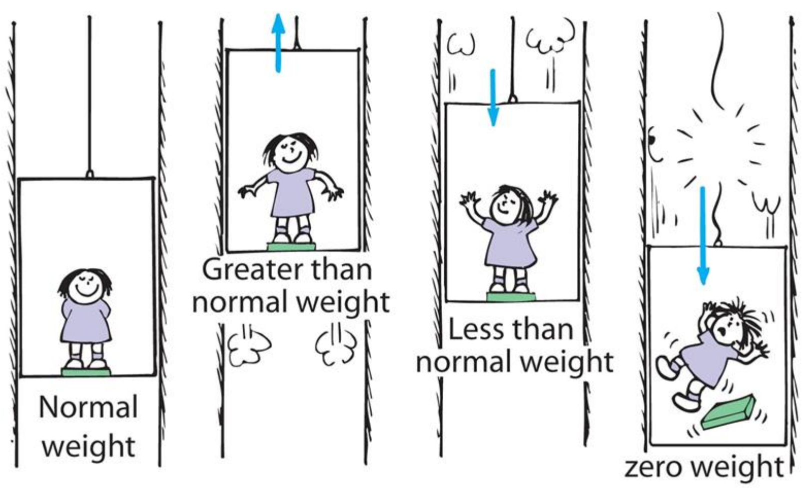
$$w = m \cdot g$$

$$w = 1200kg \cdot 9.8 \, {}^{m}\!/_{s^2} = 11{,}760N$$

Example 2: A spaceship landed on Mars. The mass of the spaceship on Earth is 30,000 kg. Calculate the weight of the spaceship on Mars.

$$w = m \cdot g$$

$$w = 30{,}000kg \cdot 3.70 \, {}^{m}\!/_{s^2} = 111{,}000N$$



Weight or heaviness can only be felt if the object is supported underneath by a surface (the normal force), such as the solid surface of a planet or a building's floor, that pushes back against the force of gravity. If the object is unsupported, then the object has the sensation of being weightless because there is no surface pushing up. The object still has weight because the object has mass (m) is acted upon by the gravity field in the downward direction (g).

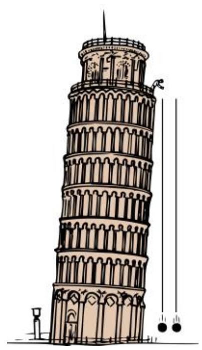
PART 4: FALLING OBJECTS

Learning Targets

1. Students will calculate freefall velocity and freefall distance/height when given time and initial velocity.
2. Students will compare and contrast the velocity and acceleration of falling objects of different or similar masses, of different or similar sizes and shapes, and in different gravity settings.
3. Students will explain Galileo's gravity experiments with regard to falling objects.
4. Students will describe how air resistance affects falling objects.

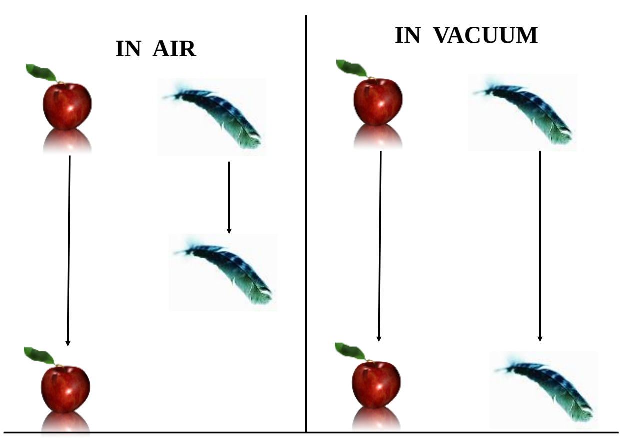
Freefall is the natural downward motion of falling objects under the influence of gravity. Earth's gravity field, like the gravity field of any planet or moon, pulls objects downward toward its surface. In the absence of all other forces (e.g., air resistance, friction, wind) on Earth, objects will naturally freefall downward toward the Earth's surface and freefall velocity will increase with time due to the downward acceleration of Earth's gravity field.

The downward acceleration in Earth's gravity field is $g \approx 9.81\ m/s^2$. This means that for every second an object freefalls (without interference from other forces, the object's downward velocity increases by -9.8 m/s. Masses of objects are irrelevant. ***Earth's gravity is omnipresent*** and affects all objects equally in the *absence of air resistance's effects or other opposing forces*.



Galileo demonstrated that if two objects of comparable sizes and shapes (smooth spheres), but vastly differing masses (1 kg vs. 10 kg), are released at the same time from the same height above the ground, they should impact the ground at the same time. Because our atmosphere is composed of ***air*** (a gas mixture), ***air resistance*** will surely affect the downward motion of freefalling objects.

Lightweight objects with ***large surface areas***, like feathers and paper, are greatly affected by air resistance and will float and drift downward rather than fall straight down because they catch air molecules and air currents. Conversely, smoother and streamlined objects tend to be affected lesser by air resistance. As a thought experiment, if all air was removed from the room, such as in a vacuum, all objects regardless of surface area and mass would freefall at the same acceleration. No air means no air resistance.



For example, an apple and a feather when released from the same height above the floor at the same time should impact the ground at the same time despite all of their physical differences.

Calculating Freefall Velocity

Freefall velocity (vertical motion)

$$v_f = v_o - g \cdot t$$

v_f = freefall velocity (m/s)
v_0 = initial velocity (m/s)
g = 9.8 m/s^2 (acceleration in Earth's gravity field)
t = time to fall (s)

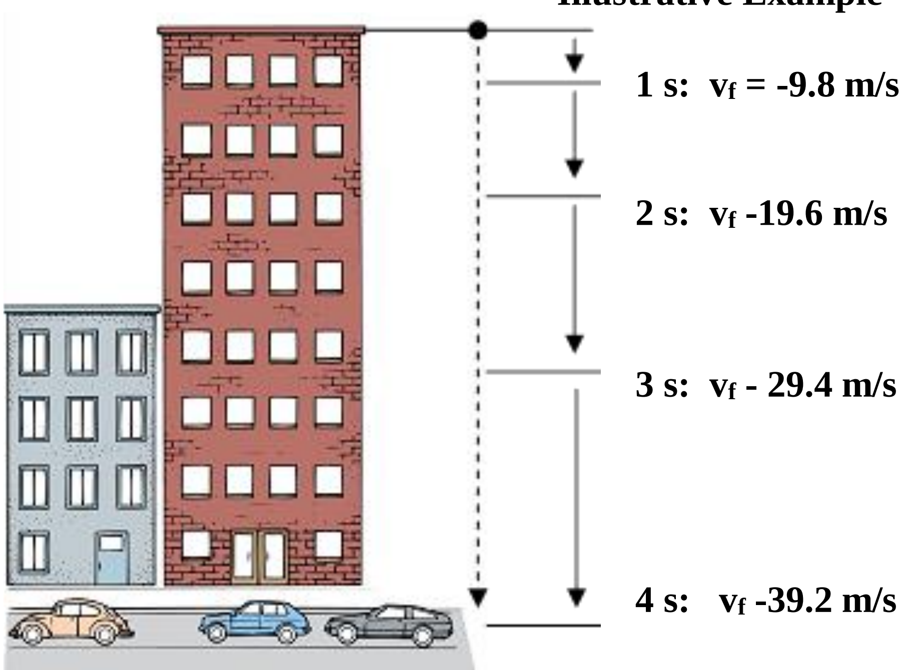
Final velocity (linear motion)

$$v_f = v_o + a \cdot t$$

v_f = final velocity (m/s)
v_0 = initial velocity (m/s)
a = acceleration (m/s^2)
t = time (s)

In Unit 1 Motion, the kinematic equation $v_f = v_o + a \cdot t$ was used to calculate the final velocity of a linearly moving object that underwent acceleration. The kinematic equation used to calculate how fast an object freefalls is almost identical, $v_f = v_o - g \cdot t$. The variable g in the freefall velocity equation is an acceleration (like a in the linear final velocity equation), it is the acceleration in Earth's gravity field. Freefall velocity is negative because gravity is direction motion in the down direction (down is negative). If air resistance is negligible, for every second that the object freefalls, the freefalling object's velocity will increase by -9.8 m/s. The initial velocity, v_0, would be 0 m/s if the object is released from a stationary position. It would be positive velocity if the object was initially thrown upward. It would be negative velocity if the object was initially thrown downward.

Illustrative Example



The figure above shows how velocity of a freefalling object changes with increasing freefall time. The object was released (v_0 = 0 m/s). The object accelerates because the velocity is getting faster. For every additional second that the object fell, its downward velocity increased by -9.8 m/s because Earth's gravity field is accelerating matter downward at 9.8 m/s^2.

Calculating changes in vertical position (Freefall distance)

Freefall distance or change in height (vertical motion)

$$\Delta y = v_0 \cdot t - \frac{1}{2} g \cdot t^2$$

$$y_f = y_0 + v_0 \cdot t - \frac{1}{2} g \cdot t^2$$

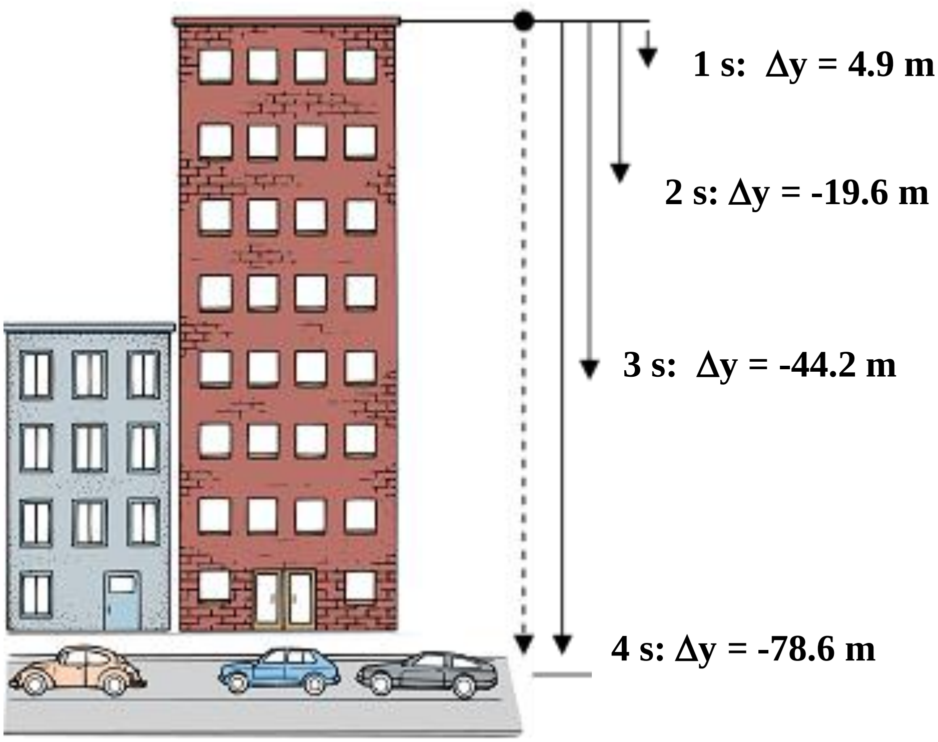
y_f = position to which the object fell (m)
y_0 = initial position at release (m)
Δy = total change in position, vertical displacement (m)
g = 9.8 m/s^2
t = time to fall (s)

Distance required to accelerate (linear motion)

$$\Delta x = v_0 \cdot t + \frac{1}{2} a \cdot t^2$$

$$x_f = x_0 + v_0 \cdot t + \frac{1}{2} a \cdot t^2$$

x_f = final position after acceleration (m)
x_0 = initial position before acceleration (m)
Δx = distance required to accelerate (m)
a = acceleration (m/s^2)
t = time (s)



If an object only freefalls, the freefall distance is negative because the displacement is downward relative to the point of release. Freefall distance increases geometrically—it becomes progressively becomes greater and greater for each second the object falls because the time variable is squared. The freefall distance is compounded as it falls because the object is falling progressively faster and faster and covering an increasing downward distance every second.

Example: A boy releases a penny from a bridge that spans a canyon. The penny freefalls for 6.0 seconds before the penny strikes the river at the bottom of the canyon. Air resistance is negligible.
(a) Calculate the freefall velocity of the penny at the instant of impact.
(b) Calculate how far the penny fell.

(a) The freefall velocity of the penny after falling for 8.5 consecutive seconds is -58.8 m/s.

$$v_f = v_0 - g \cdot t$$
$$v_v = 0\,{}^{m}\!/_{s} - 9.8\,{}^{m}\!/_{s^2} \cdot 6.0s = -58.8\,{}^{m}\!/_{s}$$

(b) The freefall distance of the penny after falling for 6.0 consecutive seconds is -176.4 m. The height of the bridge deck was 176.4 meters above the river.

$$\Delta y = v_0 \cdot t - \frac{1}{2} g \cdot t^2$$
$$\Delta y = \left(0\,{}^{m}\!/_{s} \cdot 6.0s\right) - \left(\frac{1}{2} \cdot 9.8\,{}^{m}\!/_{s^2} \cdot (6.0s)^2\right) = -176.4m$$

Example: A man is on the roof of a building. He throws a rock straight up into the air with an initial velocity of 5.0 m/s. The rock's time of flight is 7.2 seconds before it hits the sidewalk below.
(a) Calculate the freefall velocity of the rock at the instant of impact.
(b) Calculate the height of the building.

(a) The freefall velocity of the rock when it impacts the ground is -65.6 m/s.

$$v_f = v_0 - g \cdot t$$
$$v_v = 5.0\,{}^{m}\!/_{s} - 9.8\,{}^{m}\!/_{s^2} \cdot 7.2s = -65.6\,{}^{m}\!/_{s}$$

(b) The height of the building is 218.0 m above the sidewalk. The rock fell a total displacement of -218.0 m.

$$\Delta y = v_0 \cdot t - \frac{1}{2} g \cdot t^2$$
$$\Delta y = \left(5.0\,{}^{m}\!/_{s} \cdot 7.2s\right) - \left(\frac{1}{2} \cdot 9.8\,{}^{m}\!/_{s^2} \cdot (7.2s)^2\right) = -218.0m$$

Example: A boy throws a penny straight down with an initial velocity of -8.0 m/s from a bridge that spans a canyon. The penny's time of flight before it impacts the ground below is 7.9 seconds.
(a) Calculate the freefall velocity of the penny at the instant of impact.
(b) Calculate the height of the bridge above the canyon floor.

(a) The freefall velocity of the penny when it impacted the canyon floor was -85.4 m/s.

$$v_f = v_0 - g \cdot t$$

$$v_v = -8.0\,{}^{m}\!/_{s} - 9.8\,{}^{m}\!/_{s^2} \cdot 7.9s = -85.4\,{}^{m}\!/_{s}$$

(b) The freefall distance of the penny after falling for 7.9 consecutive seconds is -369 m. The height of the bridge deck was 369 meters above the canyon floor.

$$\Delta y = v_0 \cdot t - \frac{1}{2} g \cdot t^2$$

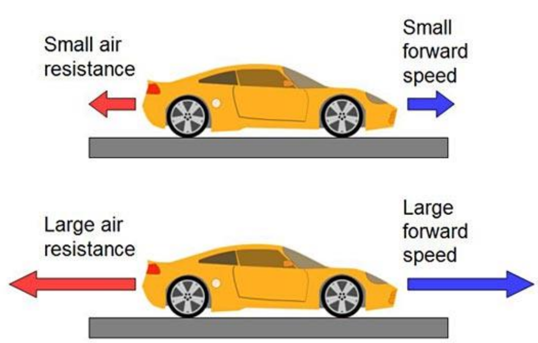
$$\Delta y = \left(-8.0\,{}^{m}\!/_{s} \cdot 7.9s\right) - \left(\frac{1}{2} \cdot 9.8\,{}^{m}\!/_{s^2} \cdot (7.9s)^2\right) = -369m$$

Air Resistance and Terminal Velocity

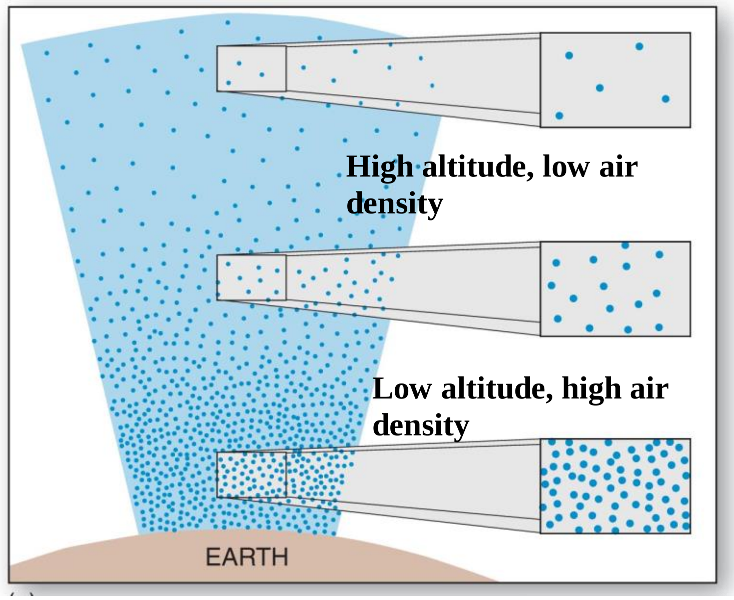
Air resistance (also knowns as ***drag force***) is the resisting force that occurs as a moving object passes through air. The moving object collides with billions and billions of air gas molecules. As each gas molecule collides with the moving object, the air molecules push back on the object. The collisions of the tiny gas molecules with the moving object converts a tiny fraction of the object's kinetic energy into heat. When billions of gas molecules collide with the object, a significant quantity of kinetic energy is converted into heat. Air resistance is proportional to the object's velocity, surface area of the object, and to the density of air.

Effect of velocity. Objects that move very fast through air accrue greater air resistance than slower moving objects.

- The faster an object moves through air, the more gas molecules in the air collide with the surface of the moving object per unit time, the greater the air resistance force.
- The slower an object moves through air, the fewer gas molecules in the air collide with the surface of the moving object per unit time, the lesser the air resistance force.



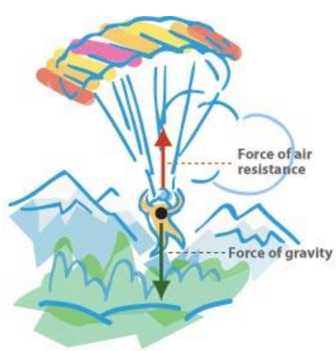
Effect of air density. The image to the right shows the unequal distribution of air in Earth's atmosphere. It is dense near the surface and gets thinner and thinner with increasing altitude. Earth's gravity pulls air molecules downward toward the Earth's surface. Air molecules at the Earth's surface are very are very close together. The air gets "thinner", or less dense, as altitude increases. At the cruising altitudes of commercial jets (8000-10,000 m above sea level), the air molecules are fewer and are spaced very far apart, which reduces the effect of air resistance.

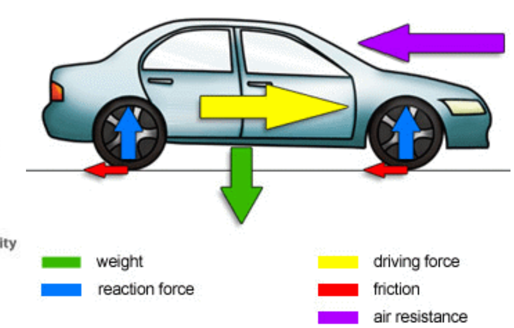


Airplanes fly at higher altitudes to avoid buildings and mountains. They also fly at higher altitudes (higher in the sky) because air density is much lower at higher altitudes than at lower altitudes.

- The denser the air (greater the air pressure), the more collisions between the object and air molecules per unit time because air molecules are more abundant and very close together, the greater the air resistance force.
- The thinner the air (lesser air pressure), the fewer collisions between the object and air molecules per unit time because air molecules are fewer and spaced far apart, the lesser the air resistance force.

Effect of surface area. Parachutes are large fabric canopies that slow objects that move through air. Parachutes "catch" air molecules. Parachutes are effective because they have a very large surface area. Objects that have greater surface areas collide with more air molecules than objects with narrower or smaller surface areas. Automobiles with higher profiles experience more air resistance force than streamlined or aerodynamic profile automobiles. The reduction of air resistance in more aerodynamic profile automobiles tends to increase fuel efficiency because more energy is put toward motion as kinetic energy and lesser energy wasted as heat.

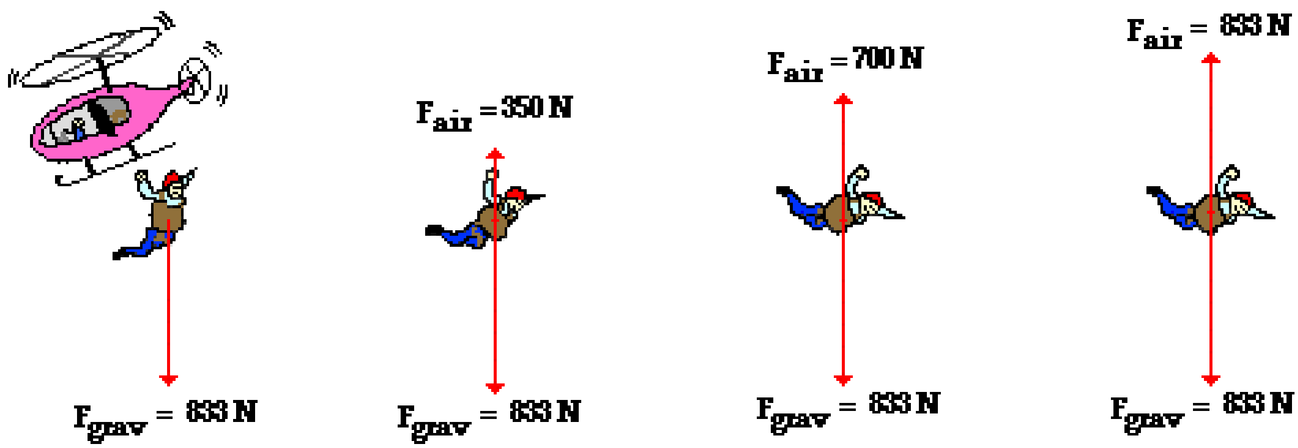




Air Resistance and Falling Objects

In the absence of air, objects will continue to accelerate in the downward direction toward the Earth's surface gaining and additional -9.8 m/s of velocity for each consecutive second the object falls. As stated before, Earth's atmosphere contains air which creates ***air resistance*** or ***drag***—a force similar to friction that acts in the opposite direction of the motion of objects moving through dense layers of gas molecules. For a freefalling object (falling down), air resistance force acts in the up direction.

When an object falls through air, and air resistance begins to act upon the freefalling object, the object continues to gain speed as it falls, however, it gains speed at a lesser and lesser acceleration because air resistance increases as the object's velocity gets greater. Freefalling objects in Earth's atmosphere, even streamlined objects, will eventually achieve a constant downward falling velocity called terminal velocity. At terminal velocity, the downward force of gravity acting upon the falling object is exactly equal in magnitude and opposite in direction to the upward push force of air resistance. This is balanced forces. The falling object will fall downward at a constant rate, no longer accelerating.

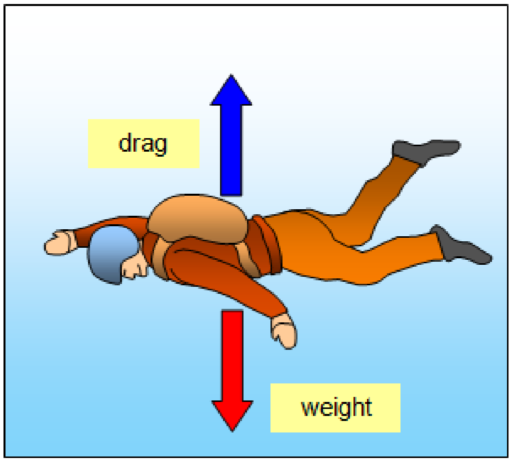


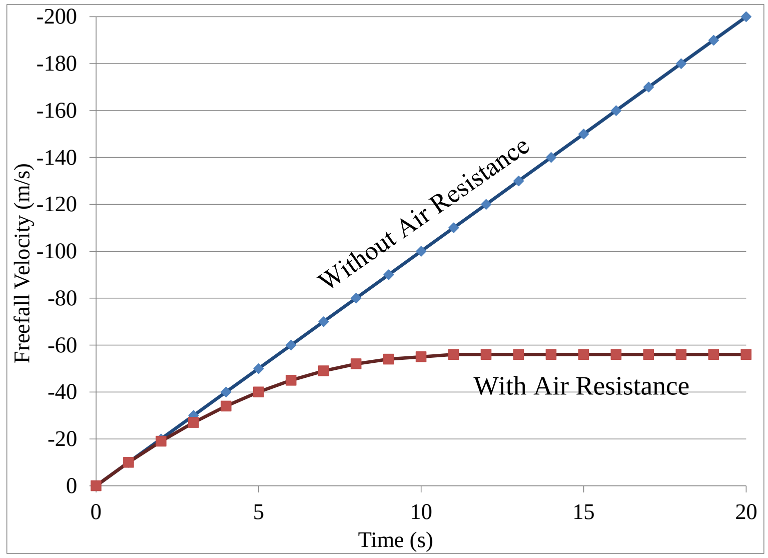
At jump, t = 0
Jumper is accelerating downward

Velocity increases with time, but at a progressively slower and slower rate as air resistance increases.

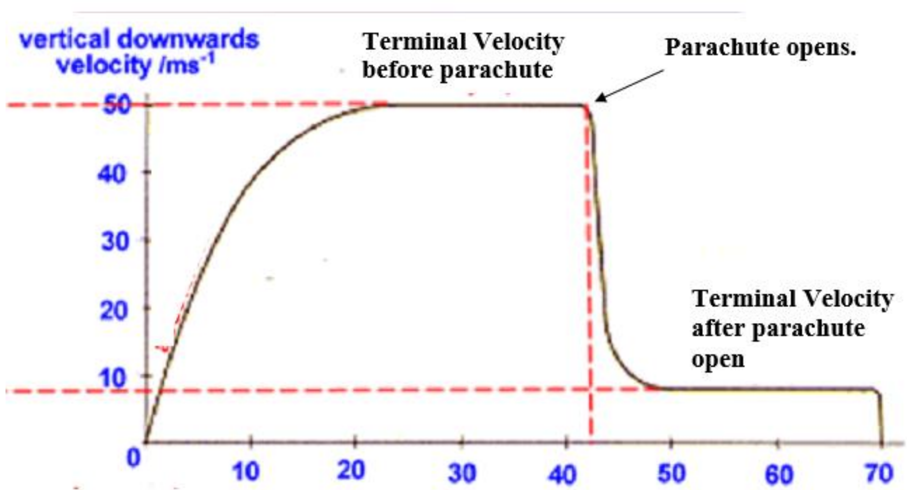
Terminal velocity. Falling at a constant velocity. No acceleration.

Terminal velocity is the maximum constant freefall velocity an object can achieve because the downward force of gravity pulling the object (weight) is equal to and opposite the upward push force of air resistance force—balanced forces. Under this balanced force condition, the object is moving the fastest and moving at a constant velocity downward, and is no longer accelerating (getting faster). The object will move at the constant terminal velocity until it impacts the ground, or if he/she deploys the parachute. The graph contrasts freefall velocity without air resistance and with air resistance. The straight line shows ideally the increasing downward velocity of an object in a vacuum or without air resistance. Note that freefall velocity will continue to get faster and faster by -9.8 m/s for each additional second it falls. Conversely, the other line shows how velocity eventually reaches a local plateau or constant value because of air resistance. As the object freefalls and freefall velocity becomes faster, air resistance increases proportionally. Eventually the freefalling object reaches terminal velocity at -59 m/s. It is still moving down at the constant velocity, but is no longer getting faster because it is affected by balanced forces.





The graph to the right illustrates what happens to a freefalling object, like paratrooper, in which the parachute is deployed during freefall. Before the parachute opens, the falling paratrooper reaches his terminal velocity. Once the parachute opens, the surface area of the parachute catches air, and creates greater air resistance. The falling paratrooper instantly slows and will come to the new terminal velocity created by the upward force on the parachute.



PART 5: PROJECTILE MOTION

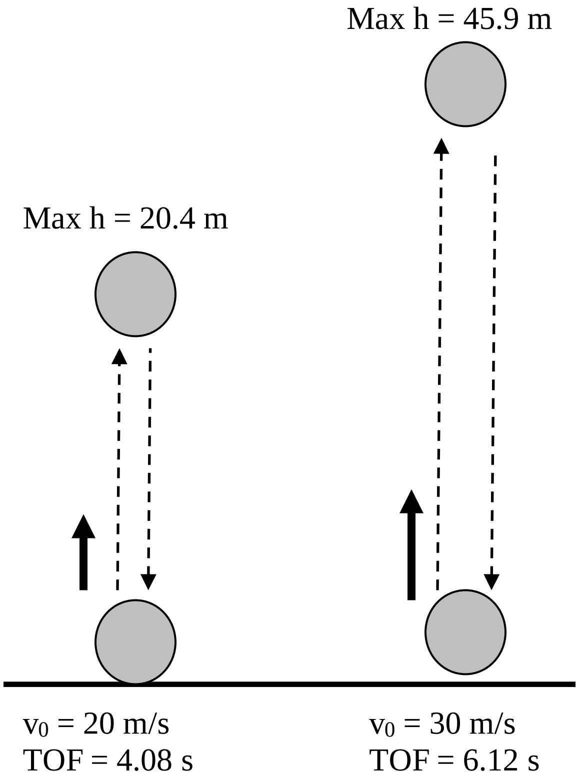
Learning Targets

1. Students will understand that projectiles are accelerated in the down direct at all points in their trajectories by the Earth's gravity.
2. Students should be able to explain how horizontal velocity and vertical velocity affect the trajectory, time of flight, and range of parabolic and horizontal projectiles.
3. Students should be able to compare and contrast different trajectories and predict which projectile should have the longest time of flight, longest range, and greatest initial launch velocity.
4. Students should be able to explain velocity vector diagrams for vertical, horizontal, and parabolic projectiles.
5. Students should be able to predict and calculate height, range, and time of flight for horizontal and vertical projectiles when provided with initial launch velocities and launch heights.

A ***projectile*** is any object that is launched and flies through the air where the only external force acting upon the airborne object is the downward pull of gravity. Bullets, arrows, rockets, cannonballs, and kicked footballs are examples of projectiles. ***When a projectile is launched, the project is ALWAYS being accelerated downward by Earth's gravity at all points during its flight.*** In other words, gravity always accelerates in the downward direction regardless if the object is moving up, moving horizontally, or moving down. A projectile's motion can be described by six important parameters.

- **Launch velocity**: How fast the object was launched into the air.
- **Launch angle**: The upward, horizontal, or diagonal angle at which the projectile was launched.
- ***Trajectory:*** The path of flight that the projectile takes through the air from launch to impact. Trajectory depends on the launch angle and the launch velocity.
- **Range**: Horizontal displacement that the projectile moves from its launch position to impact. "How far" the projectile flies away from launch.
- **Height**: How far the object is above the ground or surface at any position in its flight between launch and impact.
- **Time of flight**: The amount of time the projectile remains airborne from launch to impact.

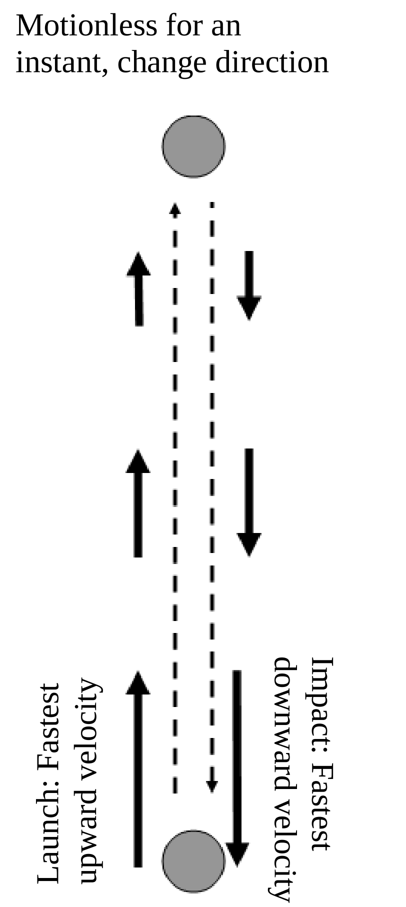
Vertical projectile motion



A **vertical projectile** is an object that is launched straight up into the air (90º launch angle), reaches its highest position, stops for an instant as it changes direction, and freefalls straight down without any horizontal movement. The trajectory is only in the vertical direction, there is no range.

The **time of flight** and the maximum **height** above the ground depends on the initial launch velocity. The faster the projectile is launched upward, the farther it will fly upward into the air (greater maximum height), the longer it will remain airborne before impacting the ground.

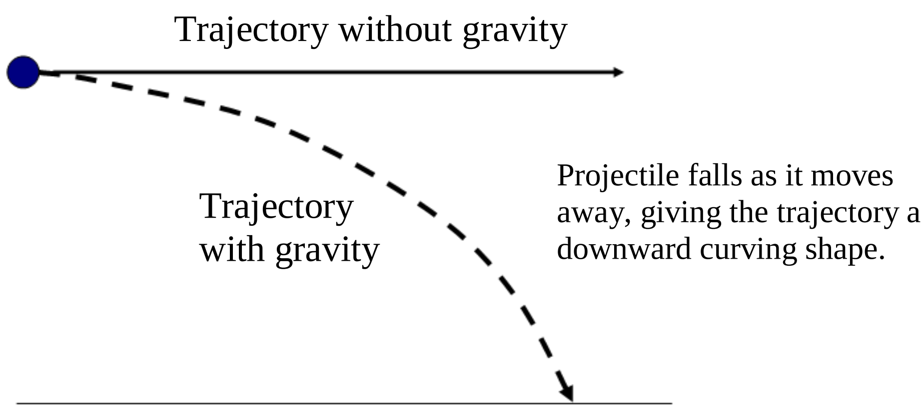
The **vector diagram** to the right shows the relative velocity of the vertical projectile as it flies through the air after it is launched. At launch, the vertical projectile is moving with its greatest upward velocity. The upward velocity becomes progressively slower the farther the projectile travels upwards (the arrows become shorter) because gravity is accelerating the projectile downward by 9.8 m/s^2 (opposite direction of motion). At its highest point above the ground (position 2), the vertical projectile is instantaneously motionless—a velocity of 0 m/s—and changes direction from upward to downward. After changing direction, the projectile freefalls to the ground. The vector arrows point in the down direction because velocity during freefall from the highest point to the ground is negative. The vector arrows become progressively longer in the down direction because the freefalling projectile is being accelerated downward at 9.8 m/s^2— freefall velocity gets faster in the downward direction. If the launch position and the impact position are at the same exact elevation, the magnitude of the upward velocity at launch is equal to the magnitude of the downward velocity at impact.



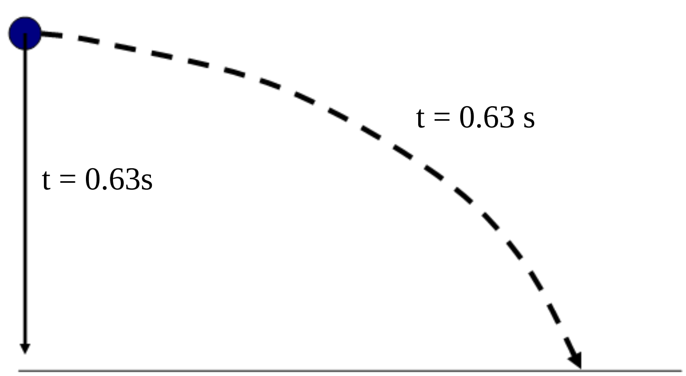
Horizontal projectile motion

A **horizontal projectile** is an object that flies through the air after being launched horizontally; the launch angle is 0º relative to the Earth's horizontal surface. Common horizontal projectiles include arrows shot from a bow, bullets fired from a rifle or handgun, and rocks launched from sling shots.

Horizontal projectiles are launched with an initial velocity that is only in the horizontal direction. This causes the projectile to move away from the place where it was launched. Once the projectile is airborne, gravity begins to affect the projectile's motion. In other words, it immediately falls towards the Earth's surface as it moves away from its launch position. The simultaneous falling and moving away, giving the horizontal projectile a downward curving trajectory (curved dashed arrow). If gravity did not affect the motion of the horizontal projectile, it would fly through the air in a straight line forever moving at its launch velocity (straight solid arrow).

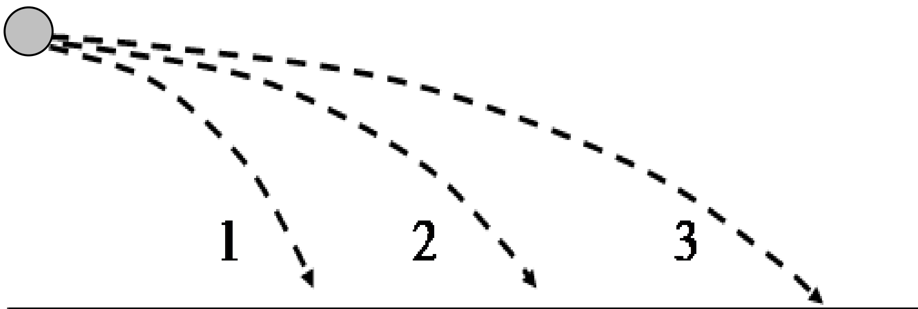


The ***time of flight*** for the horizontal projectile to impact the ground *is only dependent the height above the ground from which the horizontal projectile was launched*, and NOT the initial horizontal speed at which it was launched. The time to impact the ground after launch equals the amount of time for the project to freefall to the ground if it was simply dropped from the same height as it was launched.

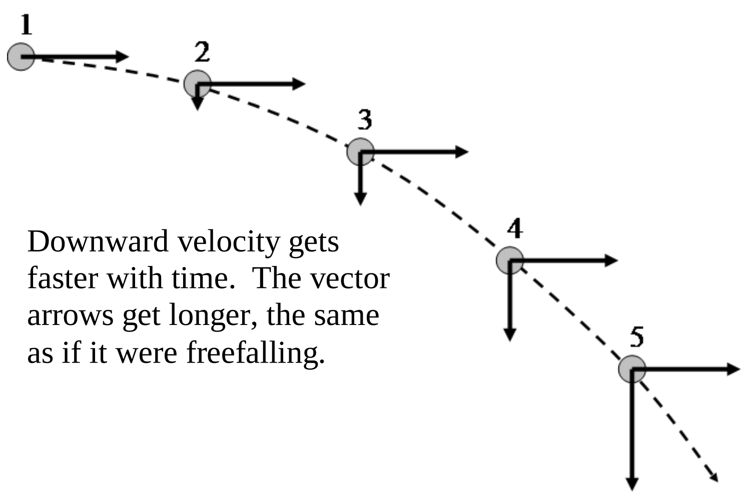


For example, if a bullet was fired horizontally from a gun positioned 2.0 meters above the ground will impact the ground in 0.63 seconds. A bullet freefalling 2.0 meters will impact the ground in 0.63 seconds. They simply land with different ranges.

The ***initial horizontal launch velocity*** controls the horizontal projectile's ***range***—how far it moves horizontally away from its launch position. The greater the initial horizontal velocity at launch, the greater the projectile's range. In the, three different projectiles were launched horizontally from the same position above the ground, but at different launch velocities.



Projectile 1 was launched with the slowest initial velocity. Projectile 3 was launched with the fastest initial velocity. Projectile 1 will have the shortest range and projectile 3 will have the longest range. Despite that projectiles 1, 2, and 3 were launched with different initial horizontal velocities, *all three projectiles will impact the ground with exactly with the same time of flight because they were launched horizontally from the same height above the ground*. They only impact the ground at different ranges because they were launched with different horizontal velocities.



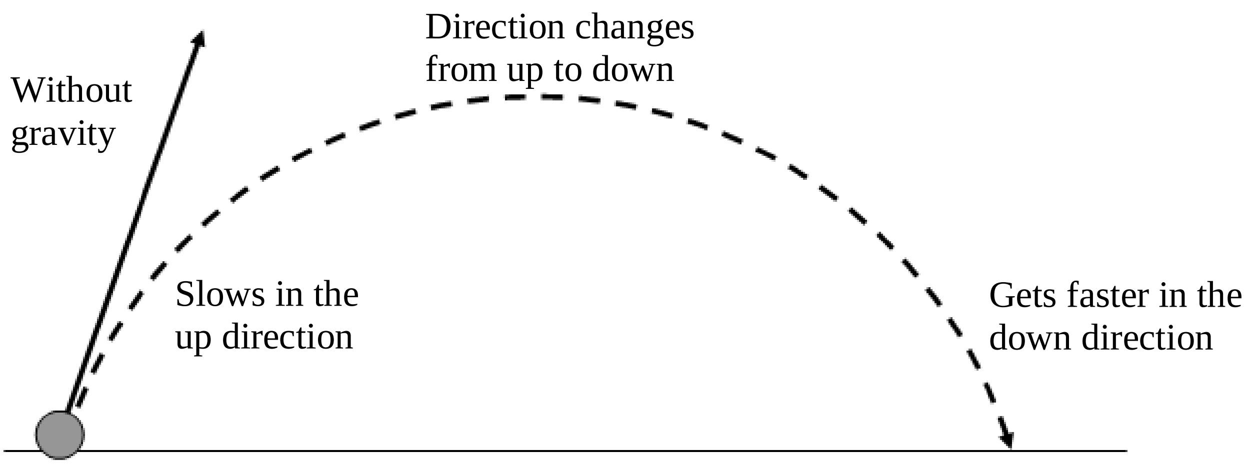
The ***vector diagram*** shows the relative velocity components of the horizontal projectile in its trajectory. The horizontal arrows represent the horizontal velocity component (how fast it moves away from the launch position through its range). The downward pointing arrows represent the downward velocity component (how fast it falls as it moves through the air).

The horizontal velocity vectors are equal length at all positions in its trajectory. Horizontal velocity never changes—the project will continue to move horizontally through its range at the same initial velocity as when it was launched.

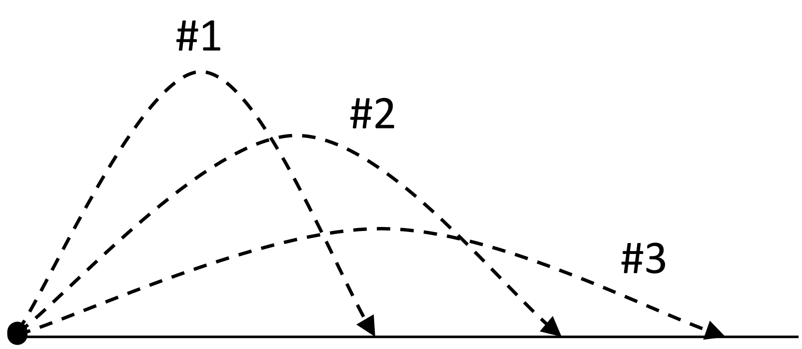
In contrast, the downward velocity vectors become progressively longer with distance away from the launch position. At launch (position 1), there is no downward pointing arrow, at launch all motion is horizontal. As the airborne projectile flies farther away from the launch position—from positions 2 to 5—the lengths of the downward pointing arrows become longer and longer. Gravity is accelerating the projectile downward. The longer the time of flight, the farther the project progressively falls, the greater the projectile's downward velocity.

Parabolic projectile motion

A **parabolic projectile** is an object that is launched at an upward angle or diagonal direction ($0^{\circ} < \theta < 90^{\circ}$) and travels through the air with a parabolic (upside down U-shape) trajectory. Parabolic projectile motion is the most common, and is the most frequently observed at sporting events: passing a football, hitting a golf ball, throwing a javelin, etc… The two factors affects parabolic projectile motion is the ***launch angle*** and ***initial launch velocity***.



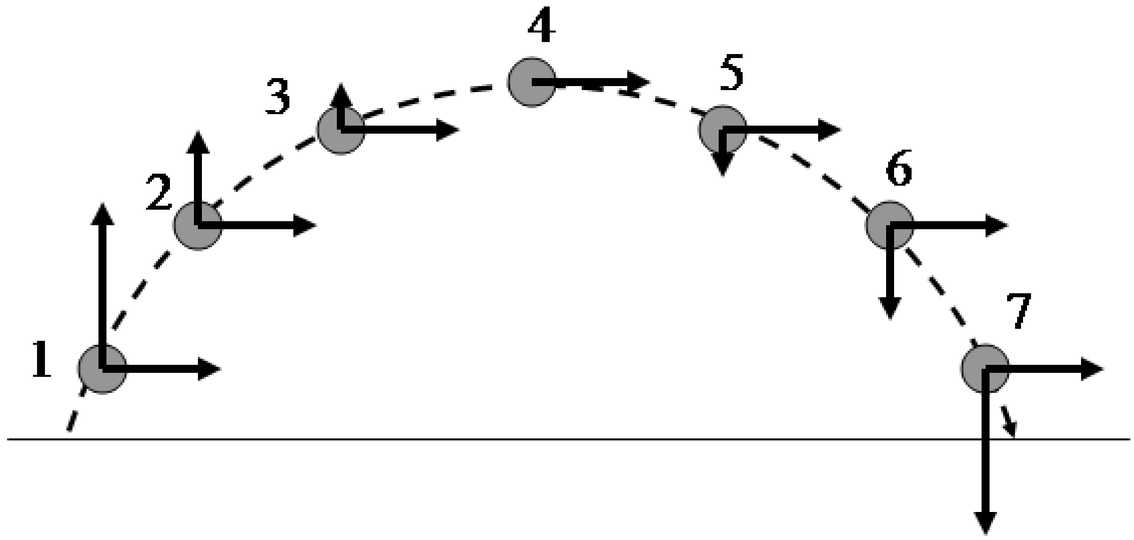
Parabolic projectiles are launched at an upward angle or at a diagonal ($0^{\circ} < \theta < 90^{\circ}$) relative to the ground. This causes the projectile simultaneously to move upward and away from its launch position. Once the projectile is airborne, gravity begins to affect the projectile's motion by accelerating the projectile in the down direction at 9.8 m/s^2 at all positions during its flight. The projectile experiences a direction change from upward to downward after reaching its highest position above the ground, giving the projectile's flight path the upside down U shape (curved dashed arrow). If gravity did not affect the motion of the horizontal projectile, it would fly through the air in a straight line forever moving at its launch velocity (straight solid arrow).



The ***time of flight***—how long the projectile remains airborne—depends only on how high above the ground the projectile flies. Projectile #1 will have the longest time of flight because its trajectory was the highest above the ground—it had the greatest height from which to fall. Projectile #3 has the shortest time of flight because its trajectory was the lowest in altitude despite that it covered the farthest range.

The **vector diagram** shows the relative vertical velocity component and horizontal velocity component of the parabolic projectile during its time of flight. The horizontal arrows represent the horizontal velocity component (how fast it moves away from the launch position through its range). The upward and downward pointing arrows represent the vertical velocity components (how fast the projectile is moving up or down under the influence of gravity as it flies through the air).

The horizontal velocity vectors are equal length at all positions. Horizontal velocity never changes—the project will continue to move horizontally through its range at the same horizontal velocity component as when it was launched. In other words, it moves away from launch laterally with the same velocity.



In contrast, the vertical velocity vectors change direction and change length with time of flight. At the instant of launch (position 1), the projectile has the greatest upward velocity. As the projectile moves away from launch, the lengths of the upward velocity vector arrows become smaller until the projectile reaches the highest position above the ground (position 4), at which vertical velocity is zero. At the highest position above the ground, the object only moves with horizontal velocity for an instant. As the projectile moves past the highest position, the projectile falls toward the Earth's surface. The lengths of the vertical velocity vectors point downward and become progressively longer until the projectile impacts the ground (at position 7) where it has the greatest downward vertical velocity.

If the parabolic projectile is launched and lands at exactly the same height (e.g., launched from the ground and impacts the ground), the launch angle is equal to the impact angle AND the magnitude of the upward vertical launch velocity component is equal to the magnitude of the downward vertical velocity component at impact.