Unit 1 Kinematics Study Guide

Concept Summary

Concept	Equation (if applicable)	Description
Scalar		A quantity that only has magnitude
Vector		A quantity that has both magnitude and direction
Frame of reference		A defined origin and axes that specify the directions (N, S, E, W; left, right, up, down) in a problem
Position		Where an object is located relative to a frame of reference; measured with the unit **meters** (m)
Distance		The total length of the path travelled; is always a positive number; is a scalar; measured with the unit **meters** (m)
Displacement	In one-dimension: $$\Delta x = x_f - x_i$$ x_f = final position x_i = initial position	The change in an object's position from its starting point to its ending point; is a vector that points from the starting point to the ending point; measured with the unit **meters** (m)
Average Speed	$$\text{avg speed} = \frac{\text{distance}}{\text{time}}$$	How "fast" an object travels; the distance an object travels divided by the length of time travelled; is a scalar; measured with the unit **meters/second** (m/s)
Average Velocity	$$\text{avg velocity} = \frac{\text{displacement}}{\text{time}}$$	An object's displacement divided by the length of time travelled; is a vector; measured with the unit **meters/second** (m/s)
Instantaneous Speed		The speed of an object at a single moment of time; equals the magnitude of the instantaneous velocity; usually referred to as just "speed"; is a scalar; measured with the unit **meters/second** (m/s)
Instantaneous Velocity		The velocity of an object at a single moment of time; usually referred to as just "velocity"; is a vector; measured with the unit **meters/second** (m/s)
Acceleration	$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t}$$ v_f = final velocity v_i = initial velocity Δt = time	The change in velocity per unit of time; occurs when an object increases or decreases its speed and/or changes the direction it is moving; is a vector; measured with the unit **meters/second²** (m/s^2) which is equivalent to "meters per second per second"
Kinematic Equations	Original forms: $$v_f = v_i + a \cdot \Delta t$$ $$\Delta x = v_i \cdot \Delta t + \frac{1}{2}a(\Delta t)^2$$ $$v_f^2 = v_i^2 + 2a \cdot \Delta x$$ Rearranged forms:	Equations that tell the relationship between initial and final velocity, displacement, time, and acceleration; are only valid when acceleration is constant

	$a = \frac{v_f - v_i}{\Delta t}$ $a = \frac{2(\Delta x - v_i \cdot \Delta t)}{(\Delta t)^2}$ $a = \frac{v_f^2 - v_i^2}{2 \cdot \Delta x}$ v_f = final velocity v_i = initial velocity a = acceleration Δx = displacement (change in position) Δt = time	
Slope	$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$ (x_1,y_1) and (x_2,y_2) are two points on the line	The "steepness" of a straight line on a graph; the change in the *y*-variable divided by the change in the *x*-variable; can be positive (uphill), negative (downhill), or zero (horizontal).
Position vs. Time Graph		A graph of motion with position on the *y*-axis and time on the *x*-axis; the slope of the line equals the velocity
Velocity vs. Time Graph		A graph of motion with velocity on the *y*-axis and time on the *x*-axis; the slope of the line equals the acceleration

Unit Conversion

To convert from one unit to another, multiply the given quantity by a conversion factor. In the conversion factor, the unit you are converting *into* goes on the top and the unit you are converting *from* goes on the bottom.

Example

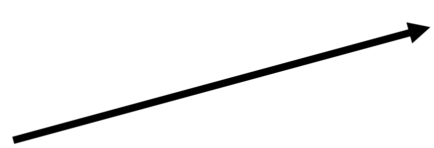
Convert 5 kilometers into meters (1 km = 1000 m).

$$5 \text{ km} \times \frac{1000 \text{ m}}{1 \text{ km}} = 5000 \text{ m}$$

Vectors and Scalars

A *scalar* is a quantity that only has magnitude. Some examples of scalars are distance, temperature, and time.

A *vector* is a quantity that has both magnitude and direction. A vector is represented by an arrow, like this:



The length of the arrow represents the magnitude and the arrow points in the direction of the vector. Some examples of vectors are velocity, acceleration, and force.

When writing a vector, the direction needs to be stated. In two dimensions, this can be done with:

- North, South, East, West
- Right, left, up, down

In one dimension, vector direction is stated with a positive (+) or negative sign (-). We define the positive and negative directions as follows:

- Positive (+): right, up, East, North
- Negative (-): left, down, West, South

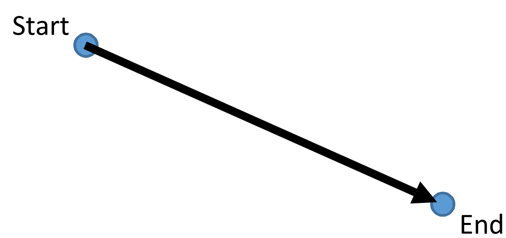
Magnitude means the amount, how big, how much.

Distance and Displacement

Position is where an object is located relative to a frame of reference. Position is represented by the variable x. When an object is in motion, its position is changing.

As an object moves along some path, *distance* is the total length of the path that is travelled. To calculate distance, just add up the lengths of the different segments of the path that is travelled. Distance is a scalar. It is always a positive number. Distance is measured with the unit **meters** (m).

As an object moves along some path, the *displacement* is the change in position from the starting point to the end point. Displacement is a vector; it has a magnitude (length) and direction. It is measured with the unit **meters** (m). To draw a displacement vector, you just draw an arrow that starts at the starting point and ends at the end point, with the arrow pointing towards the end point:



In one dimension, displacement (Δx) can be calculated with this equation, where x_f is the final position and x_i is the initial position:

$$\Delta x = x_f - x_i$$

When using this equation, make sure to plug in the correct signs. For example, if a car starts at 2 m and ends at -5 m, its displacement is

$$-5 \text{ m} - 2 \text{ m} = -7 \text{ m}$$

Note that I plugged in -5 for the final position and not just 5. The fact that the displacement is negative tells us the car ended to the left of where it began.

Example

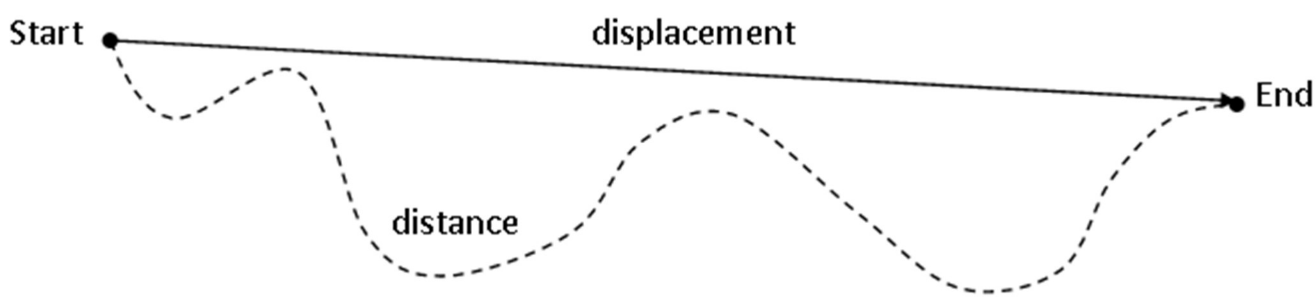
You live 2 km from the supermarket. You drive to the supermarket and back home. What is your distance and displacement?

You travelled 2 km on the way to the store and another 2 km on the way back home, so your distance is

$$2 \text{ km} + 2 \text{ km} = 4 \text{ km}$$

Your displacement is 0 km because the starting point and the end point are the same. In that case, there is no change in position from the starting point to the end point, so displacement is zero.

Example



The dotted line is the path that is travelled. The distance is the length of the dotted line. The displacement is the vector drawn from the start to the end point.

Does Distance = Displacement?

As the above examples demonstrate, the distance is generally not going to equal the magnitude of the displacement.

The one case in which distance will equal the magnitude of the displacement is when the motion is along a straight line in one direction (no backtracking).

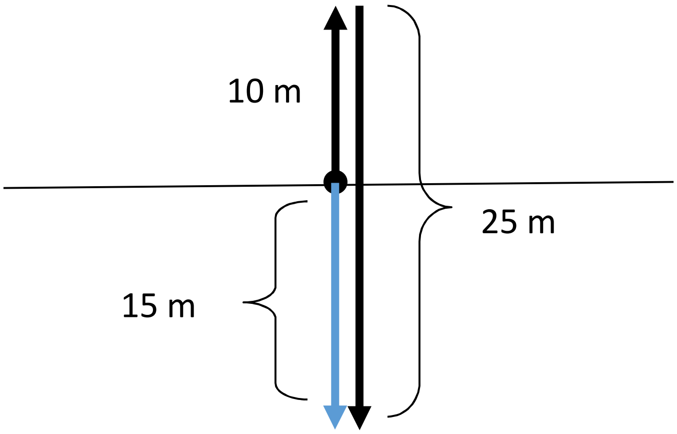
No matter how large of a distance something travels, if it returns to where it started, the displacement is zero. You can travel around the whole Earth, but if you return to where you started, your displacement is zero. *If the starting and end points are the same, displacement is zero!*

Solving Distance and Displacement Problems

If a problem asks you to find distance and displacement, follow these steps:

1. Draw a picture with arrows showing the different segments along which the object travelled. Label the length of each segment. (Note: you should start each arrow where the previous one ended, not back at the origin!)
2. Draw the displacement vector on your picture. Start at the starting point and draw your displacement vector as a straight arrow going to the end point and pointing towards the end point.
3. Add the length of the segments to find distance.
4. Use the drawing to determine the magnitude of the displacement vector and its direction.

Example



A man walked 10 meters north, turned, and walked 25 meters south.

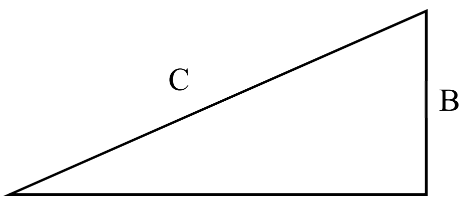
The black arrows show how the man walked (using coordinate lines for reference). The displacement vector is drawn in blue.

The man walked a total distance of 35 meters.

$$10 \text{ m} + 25 \text{ m} = 35 \text{ m}$$

The man's displacement from start to finish was 15 meters S (or -15 m).

If the problem includes motion in two directions, you will need to use the Pythagorean Theorem to calculate the length of the displacement vector.

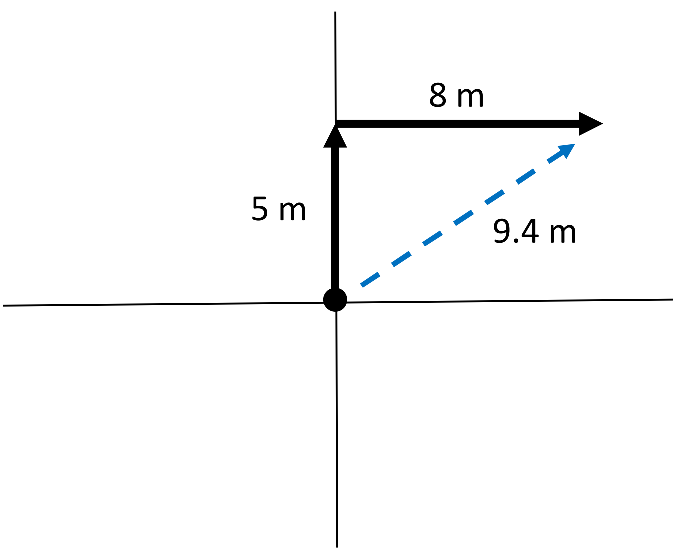


$$C^2 = A^2 + B^2$$

$$C = \sqrt{A^2 + B^2}$$

Example

A man walked 5 meters north, turned, and walked 8 meters east. The solid arrows show how the man walked (using geographic grid lines for reference). The dotted blue arrow is the displacement vector.



The man walked a total distance of 13 meters.

$$5 \text{ m} + 8 \text{ m} = 13 \text{ m}$$

The man's displacement from start to finish was 9.4 meters NE. The Pythagorean Theorem was used because the man's path of travel was in 2 directions where the displacement is the hypotenuse of the right triangle.

$$C^2 = A^2 + B^2$$

$$C^2 = (5 \text{ m})^2 + (8 \text{ m})^2$$

$$C^2 = 89 \text{ m}^2$$

$$C = \sqrt{89 \text{ m}^2} = 9.4 \text{ m}$$

Speed and Velocity

Average Velocity and Average Speed

Average velocity describes how position changes as time changes. It is the displacement divided by the time over which that displacement occurs. Average velocity is a vector. It is measured in the unit **meters/second** (m/s). The equation is

$$\text{avg velocity} = \frac{\text{displacement}}{\text{time}}$$

When using this equation, the displacement must be in meters and the time in seconds! If you are not in these units, you must first convert into them.

The average velocity equation can be manipulated using algebra into these two equations, which can be used to find displacement and the time an object is traveling:

$$\text{displacement} = (\text{avg velocity})(\text{time})$$

$$\text{time} = \frac{\text{displacement}}{\text{avg velocity}}$$

Examples

A dog runs a displacement of 3 m N in 2 s. What is its average velocity?

$$\text{avg velocity} = \frac{3\text{ m N}}{2\text{ s}} = 1.5\text{ m/s N}$$

A dog runs with an average velocity of 4 m/s E for 10 s. What is its displacement?

$$\text{displacement} = (4\text{ m/s E})(10\text{ s}) = 40\text{ m E}$$

How long did a dog run if its displacement was 20 m S and its average velocity was 2 m/s S?

$$\text{time} = \frac{20\text{ m S}}{2\text{ m/s S}} = 10\text{ s}$$

Average speed describes how "fast" an object is moving. It is the distance divided by the time over which that distance occurs. Average speed is a scalar. It is measured in the unit **meters/second** (m/s). The equation is

$$\text{avg speed} = \frac{\text{distance}}{\text{time}}$$

When using this equation, the distance must be in meters and the time in seconds! If you are not in these units, you must first convert into them.

The average speed equation can be manipulated using algebra into these two equations, which can be used to find distance and the time an object is traveling:

$$\text{distance} = (\text{avg speed})(\text{time})$$

$$\text{time} = \frac{\text{distance}}{\text{avg speed}}$$

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A dog runs with an average speed of 4 m/s for 10 s. What is its distance?

$$\text{distance} = (4\text{ m/s})(10\text{ s}) = 40\text{ m}$$

How long did a dog run if its distance was 20 m and its average speed was 2 m/s?

$$\text{time} = \frac{20\text{ m}}{2\text{ m/s}} = 10\text{ s}$$

Instantaneous Speed and Instantaneous Velocity

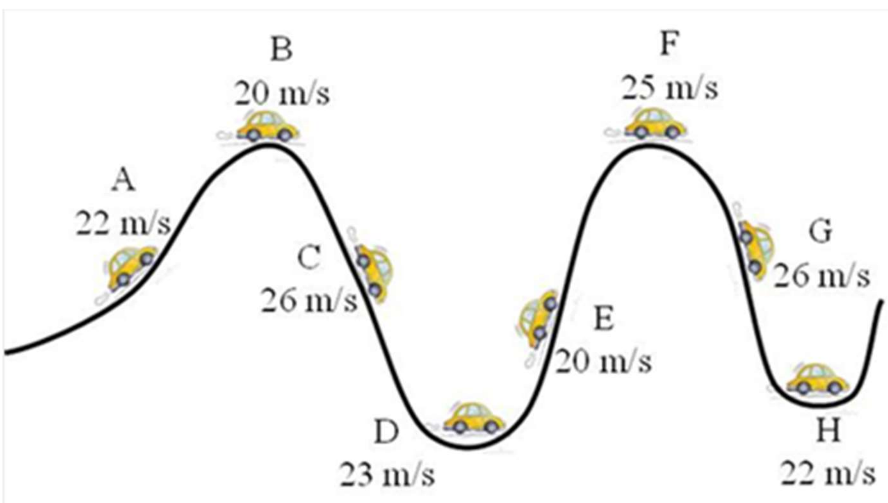
Instantaneous speed is how fast an object is moving at one moment in time. The speedometer in a car shows your instantaneous speed. It is a positive scalar and measured with the unit **meters/second** (m/s).

Instantaneous velocity is an object's velocity at one moment in time. It is a vector and measured with the unit **meters/second** (m/s).

We usually drop the instantaneous and just call them "speed" and "velocity."

The magnitude of the instantaneous velocity equals the instantaneous speed.

Example



At Points B and E the instantaneous speeds are the same (20 m/s), but the instantaneous velocities are different because the directions are different. At Points C and G the instantaneous velocities are equal because they have the same speed (26 m/s) and the same direction.

Acceleration

Acceleration describes how velocity changes as time changes. It is the change in velocity divided by the time over which that change occurs. Acceleration is a vector (it will have a magnitude and a + or – sign to designate its direction). It is measured with the unit **meters/second²** (m/s²). This unit is equivalent to saying "meters per second per second." The equation for acceleration is

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t}$$

Acceleration can occur in four ways:

1. The speed increases
2. The speed decreases
3. The direction changes
4. Speed and direction both change

Determining the Direction of Acceleration

Acceleration can be in the positive or negative direction. The effect of an acceleration on speed will depend on the direction of the velocity and the acceleration. Namely:

- Speed will *increase* if velocity and acceleration are in the *same* direction
- Speed will *decrease* if velocity and acceleration are in the *opposite* direction

In a problem, to determine the direction of acceleration:

1. Determine the direction of the velocity
2. Determine if speed is increasing or decreasing
3. If speed is increasing, acceleration is in the same direction as the velocity
4. If speed is decreasing, acceleration is in the opposite direction of the velocity

Examples

An object is moving to the left and increasing its speed. The velocity is to the left. The speed is increasing, so we know velocity and acceleration are in the same direction. Therefore, acceleration is to the left.

An object is moving to the right and slowing down. The velocity is to the right. The speed is decreasing, so we know velocity and acceleration are in opposite directions. Therefore, acceleration is to the left.

Kinematic Equations

The *kinematic equations* are equations that relate initial and final velocities, displacement, acceleration, and time. They are only true when acceleration is constant. Each can be modified using algebra into a form that we can use to calculate acceleration:

Original Form	Modified Form
$v_f = v_i + a \cdot \Delta t$	$a = \frac{v_f - v_i}{\Delta t}$
$\Delta x = v_i \cdot \Delta t + \frac{1}{2} a (\Delta t)^2$	$a = \frac{2(\Delta x - v_i \cdot \Delta t)}{(\Delta t)^2}$
$v_f^2 = v_i^2 + 2a \cdot \Delta x$	$a = \frac{v_f^2 - v_i^2}{2 \cdot \Delta x}$

Note: The first two rows are the most important of the kinematic equations to learn.

Use the kinematic equation that matches the information you know. For example, if you are looking for acceleration and you already know the displacement, initial velocity, and the time, you can use this equation:

$$a = \frac{2(\Delta x - v_i \cdot \Delta t)}{(\Delta t)^2}$$

Solving Problems Using Kinematic Equations

This is the strategy you should use to solve problems using kinematic equations:

1. Read the problem carefully. Pick out the information that is being given to you and write it down.
2. Determine what the question is asking you to find.
3. Pick the kinematic equation that matches what you are trying to find and what information you already know.
4. Plug the given information into the equation and calculate the final answer.

Example

A car is traveling at 27 m/s. The car slows down (but not to a stop) over 5 seconds. During these 5 seconds, the car travels 100 m.

- What was the car's acceleration?

1. I have to figure out what information is being given to me. It says the car is traveling at 27 m/s. That is before it started to slow down, so it has to be the initial velocity. The car slows down over 5 seconds, so that is the time. While it is slowing down, the car travels 100 m, so that is the displacement. This is what is given to me:

$$v_i = 27 \text{ m/s}$$
$$\Delta t = 5 \text{ s}$$
$$\Delta x = 100 \text{ m}$$

2. What do I have to find? It wants the car's acceleration:

$$a = ?$$

3. Which equation can I use to calculate a when I know v_i, Δt, and Δx? The only kinematic equation that matches these is this one:

$$a = \frac{2(\Delta x - v_i \cdot \Delta t)}{(\Delta t)^2}$$

4. Now I am ready to plug in to get my answer:

$$a = \frac{2\,(100 - 27 \cdot 5)}{(5)^2} = -2.8 \text{ m/s}^2$$

- What is the car's velocity at the end of the 5 seconds?

1. Now I know the acceleration, so I can add that to my list of known information:

$$v_i = 27 \text{ m/s}$$
$$\Delta t = 5 \text{ s}$$
$$\Delta x = 100 \text{ m}$$
$$a = -2.8 \text{ m/s}^2$$

2. What do I have to find? It wants the car's final velocity:

$$v_f = ?$$

3. Which equation can I use to find v_f when I know v_i, Δt, Δx, and a? This one matches what I know and am trying to find:

$$v_f = v_i + a \cdot \Delta t$$

4. Now I am ready to plug in to get my answer:

$$v_f = 27 + (-2.8)(5) = 13 \text{ m/s}$$

Position vs. Time and Velocity vs. Time Graphs

The *slope* of a line is how "steep" the line is. It is the change in the y-variable divided by the change in the x-variable. Slope can be positive (going uphill), negative (going downhill), or zero (flat, horizontal). For two points on a line, (x_1,y_1) and (x_2,y_2), the slope of the line is given by

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

A *position vs. time graph* is a graph that has position on the y-axis and time on the x-axis. The slope of a line on one of these graphs is

$$m = \frac{\Delta y}{\Delta x} = \frac{\Delta(\text{position})}{\Delta(\text{time})} = \frac{\Delta x}{\Delta t} = v$$

Therefore, **the slope of a position vs. time graph equals the velocity**.

A *velocity vs. time graph* is a graph that has velocity on the y-axis and time on the x-axis. The slope of a line on one of these graphs is

$$m = \frac{\Delta y}{\Delta x} = \frac{\Delta(\text{velocity})}{\Delta(\text{time})} = \frac{\Delta v}{\Delta t} = a$$

Therefore, **the slope of a velocity vs. time graph equals the acceleration**.

Refer to the handout from class about graphs for information and examples about using P vs. T and V vs. T graphs to determine information about the motion.

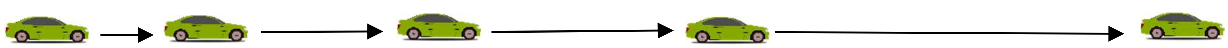
Motion Diagrams

If an object moves at constant velocity, the object moves equal distances in equal times. The spacing between the positions of the moving object at regular time intervals also remains constant. If an object moves at constant velocity, it is not accelerating; acceleration is zero.

Constant velocity

When objects accelerate, the velocity will increase (get faster) or decrease (get slower) with time. The spacing between the positions of the moving object at regular time intervals will get increasingly greater (as velocity increases) or get shorter (as velocity decreases).

Increasing velocity (getting faster)



Decreasing velocity (getting slower)